Small binary asteroids and prospects for their observations with Gaia

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ABSTRACT

We studied a capability of Gaia to detect binary systems among small asteroids (with diameters \( \leq 10 \) km) by observing their photocenter oscillation. The closest binary systems with orbital periods about 1 day or shorter show mostly a too low amplitude of the photocenter oscillation and Gaia will not be able to detect most of them with its expected astrometric performance. Wider binaries with orbital periods on an order of several days or longer shall be detectable with their amplitudes of the photocenter oscillation on an order of 10-times greater than the expected standard uncertainty of their Gaia astrometric measurements. A confusion of binaries with slowly rotating asteroids that show a rotational photocenter variation of a similar magnitude will not be significant unless the satellite is small or very large; in the range of medium-distance binaries (with the orbital periods on an order of several days), we shall be able to uniquely distinguish binaries with the diameter ratio \( D_2/D_1 \) between \( \sim 0.1 \) and \( \sim 0.95 \). Gaia will be the first survey to sample the largely unknown population of medium-distance binary systems among small main-belt asteroids where the present detection techniques (photometric and AO observations) are inefficient. A combination of Gaia binary observations with measurements with other techniques will be needed to eliminate existing degeneracy of the astrometric binary detection and to provide unique estimates of parameters of the binary systems.

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1. Introduction: binary systems among small asteroids

There exists a significant population of binary systems among asteroids with diameters \( \leq 10 \) km. Binary systems were found among near-Earth asteroids (Pravec and Hahn, 1997; Pravec et al., 1998, 2000; Mottola and Lahuella, 2000; Margot et al., 2002; Pravec et al., 2006) as well as among small main-belt asteroids (Pravec and Harris, 2007; references therein). A fraction of binaries with a secondary-to-primary mean diameter ratio \( D_2/D_1 \geq 0.18 \) among near-Earth asteroids (NEAs) larger than 0.3 km was estimated to be 15 \( \pm 4\% \) (Pravec et al., 2006). A similar fraction of binaries was estimated among small main-belt asteroids (MBAs) with \( D=3–8 \) km as well (Pravec et al., 2012).

Several interesting properties of NEA and MBA binaries were found by photometric and radar studies. One of the most important findings was that the binaries have a total angular momentum content close to the critical limit for a single body in gravity regime (Pravec and Harris, 2007). Primary components of the binaries were found to have near-spherical, top-like shapes (Ostro et al., 2006; Shepard et al., 2006; Pravec et al., 2006; Taylor et al., 2008; Benner et al., 2010). Secondary rotations are mostly synchronous with the orbital motion. Eccentricities of mutual orbits between binary components are small and the orbit planes appear close to primary equatorial planes (e.g., Ostro et al., 2006; Scheirich and Pravec, 2009; Pravec et al., 2012). Albedos of the components of a binary system are similar to within 20% (Pravec et al., 2006), consistent with being of a similar composition. Binaries appear to be ubiquitous; they were found everywhere (in all asteroid zones, groups and among all major taxonomic classes) we searched thoroughly enough. There were found also ternary systems among NEAs as well as MBAs (Brozovic et al., 2011; Marchis et al., 2008). A special case of a “quadruple” system (3749) Balam having a primary, a close satellite, a distant satellite, and a separated component (paired asteroid) was found by Vokrouhlický (2009), see references therein.

Binary systems among small asteroids (primary diameters \( D_1 \leq 10 \) km) appear to form from parent bodies spinning at a critical rate by some sort of fission or mass shedding process (Scheeres, 2007; Pravec and Harris, 2007; Walsh et al., 2008). A mechanism to spin the parent asteroid up to its critical rotation frequency is provided by the Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect (e.g., Bottke et al., 2006). Gravitational interactions of strengthless asteroids during close encounters with the terrestrial planets cannot be a primary mechanism of the formation of the binaries, but it may affect properties of the NEA part of the binary population (Walsh and Richardson, 2006).

Though we have found some interesting properties of small asteroidal binary systems and developed initial theories for their origin and evolution during the past decade, getting a better
knowledge of the binary asteroids will require to get a more complete description of the binary system. Present methods of detecting binary systems among small asteroids – the photometric, radar, and adaptive optics techniques – have significant selection effects. With radar, we can efficiently study only NEA binaries during their close approaches due to the echo strength decreasing with the 4th power of distance from the radar. A probability of binary detection with the photometric method decreases with increasing separation between binary components, allowing an efficient study of binaries with orbital periods not longer than a few days. Present adaptive optics (AO) systems have a resolution limit of 0.1–0.2 arcseconds, depending on a magnitude difference between the two bodies (with the higher value for the resolution limit applying to binaries with greater magnitude difference and hence size ratio between the components).

In Fig. 1, we plot estimated orbital period vs primary diameter data for known binaries from our database (Pravec and Harris, 2007, updated, available at web page http://www.asu.cas.cz/~asteroid/binastdata.htm). Points to the right from the dashed line which gives the approximate AO resolution limit are binaries detected with direct imaging. Points to the left from the dashed line are binaries observed by the photometric and radar technique. In the primary size range 1–10 km, there is apparent a gap between the points for close binary systems with orbital periods mostly in the range 10–50 h that were detected by the photometric technique and the points for wide binaries with orbital periods around 1000 h or longer that were detected by the AO technique. This Hic sunt leones range of binary parameters is where neither of the present binary asteroid detection methods is efficient; the medium-distance binary asteroids with orbital periods around 1000 h or longer that were detected by the AO technique and the points for wide binaries with orbital periods mostly in the range 10–50 h that were detected by the photometric method decreases with increasing separation between the points for close binary systems with orbital periods.

2. Binary asteroid detection through photocenter oscillation

The center of mass and the center of light of a binary system are

\[
\mathbf{r}_g = \frac{M_1 \mathbf{r}_1 + M_2 \mathbf{r}_2}{M}
\]

and

\[
\mathbf{r}_p = \frac{l_1 (\mathbf{r}_1 + \delta \mathbf{r}_1) + l_2 (\mathbf{r}_2 + \delta \mathbf{r}_2)}{l} = \frac{l_1 \mathbf{r}_1 + l_2 \mathbf{r}_2}{l} + \frac{l_1 \delta \mathbf{r}_1 + l_2 \delta \mathbf{r}_2}{l}
\]

respectively, where \(M, l, \mathbf{r}_1, \delta \mathbf{r}_i\) are a mass, a light flux, a gravity center position vector, and a vector of apparent disk’s photocenter offset from center of mass of the \(i\)th component, \(M = M_1 + M_2\) is the total mass and \(l = l_1 + l_2\) is the total light of the system. The second fraction on the right side is a correction term for the offsets of the photocenters of the apparent disks of the components; we will denote it with \(\mathbf{C}_p\) hereafter.

The center of light (“photocenter”) is generally displaced from the center of gravity. The displacement vector is

\[
\mathbf{\Delta r} = \mathbf{r}_p - \mathbf{r}_g = \left(\frac{l_1}{l_2} \frac{M_2}{M}\right) \mathbf{r} + \mathbf{C}_p,
\]

where \(\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1\) is a radius vector from the center of the primary to the center of the secondary.

In a case of spherical homogeneous components with the same density, albedo and a magnitude of the phase effect, the photocenter displacement vector is

\[
\mathbf{\Delta r} = [(1 + X^{-2})^{-1} - (1 + X^{-3})^{-1}] \mathbf{r} + \mathbf{C}_p,
\]

where \(X = D_2/D_1\) is a ratio of diameters of the binary components.

The radius vector circulates with a mutual orbital period of the binary system, and so does \(\mathbf{\Delta r}\) around the system’s center of mass. This periodic motion of the photocenter around the center of mass, which itself moves around the sun on the asteroid’s heliocentric orbit, can be detected with accurate astrometric observations.

3. Detectability of the photocenter oscillation

In Fig. 2, there is plotted a magnitude of the relative photocenter displacement \(\mathbf{\Delta r}/\mathbf{r}\) as function of the diameter ratio \(X\) for a case of \(\mathbf{C}_p = 0\); the function is the term in brackets in Eq. (4). The photocenter displacement reaches the maximum \(\max (|\mathbf{\Delta r}/\mathbf{r}|) \approx 0.0897\) at \(X = 0.536\). The plot shows that a relatively high amplitude of the photocenter oscillation occurs in systems with satellites about half the primary size, while systems with small satellites or with satellites approaching the size of the primary (“double asteroids”) show much smaller amplitudes of the photocenter oscillation. The y-axes on the right side of Fig. 2 show corresponding \(\mathbf{\Delta r}\) in kilometers for an example of a typical small binary with the primary diameter of 4 km and the radius of the mutual orbit \(r = 10\) km, and the amplitude of the projected displacement vector in milliarcseconds when the binary is observed at a distance of 2.3 AU and at solar elongation 90°. A sub-milliarcsecond astrometric accuracy is needed to detect the photocenter oscillation in a typical small main-belt binary.

An expected astrometric accuracy of Gaia, like for any properly designed observational project, depends on an apparent magnitude of observed object. Table 1 gives the expected standard uncertainty \(\sigma\) of Gaia single-epoch measurement of a star with the apparent \(V\) magnitude from 12 to 20; for slowly moving main belt asteroids, the accuracy is not expected to be significantly degraded. For our study, we represent the dependence of \(\sigma\) on \(V\)

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1 An alternate form of the factor in brackets in Eq. (4) is \(|(1 + X^{-2})^{-1} - (1 + X^{-3})^{-1}|\).
with following formula:

$$\sigma = 0.15 \times 10^{0.2(V - 16)}$$, \hspace{1cm} (5)

where $\sigma$ is in milliarcseconds (mas). The $\sigma(V)$ function is plotted in Fig. 3. Note that the factor of 0.2 in the exponent corresponds to $\sigma$ being inversely proportional to the square root of light flux from the object.

To investigate a capability of Gaia to observe small binary asteroids through the oscillations of their photocenter, we used data from our database of parameters of known asteroid binaries (see Section 1) and for each binary with the primary diameter $D_1 < 20$ km, we computed a ratio between its amplitude of the photocenter oscillation $A_{pc} \equiv \max(\Delta r)$ and the uncertainty of the Gaia astrometric measurement $\sigma(V)$ for asteroid’s apparent $V$ as seen from the Earth when the asteroid is at quadrature (solar elongation $90^\circ$) and with the heliocentric distance equal to the semi-major axis ($a_h$) of its heliocentric orbit. Four known binaries on Aten-type orbits were omitted as their solar elongation is $<90^\circ$ when their heliocentric distance is equal to $a_h$. We plotted resulting data $A_{pc}/\sigma(V)$ vs the orbital period $P_{orb}$ for 77 binaries in our sample in Fig. 4.

Most of the closest asteroid binaries with orbital periods less than 1 day have $A_{pc}/\sigma(V) \lesssim 1$, thus, they will be impossible or difficult to detect with Gaia unless a high number of points is obtained to beat down the noise. Close-ish binaries with orbital periods in the range 1–2 days have $A_{pc}/\sigma(V) \sim 1$ or greater, so they should be barely detectable. The four wide binaries ($P_{orb}$ of tens of days) with satellites detected with AO observations will be easily detectable as they have $A_{pc}/\sigma(V) > 10$ or greater. Most interesting should be Gaia’s work in the *Hic sunt leones* range of asteroid binaries with medium-distance satellites with $P_{orb}$ on an order of several days.

An issue that is beyond the scope of this initial study is a question of whether the number of Gaia measurements and their time distribution – it is estimated that there will be several tens of epochs during the five year duration of the Gaia mission for a typical main belt asteroid – will allow us to get a unique solution for the mutual orbit of the binary components. A model of the
binary orbit, considering the degeneracy mentioned in Section 4.1 and assuming zero orbital precession, has seven free parameters, so there should be a sufficient number of degrees of freedom to solve both for the heliocentric orbit of the center of mass of the system and the mutual orbit of the binary components. A question of whether and in what conditions we will get a unique solution, especially for the orbital period where there could be multiple solutions due to the sparse sampling of the observations, will be studied with simulations in a future paper.

4. Limitations of binary detection through photocenter oscillation

4.1. Degeneracy of the astrometric detection

With observations of the photocenter oscillation only, we cannot separate the components’ distance $r$ and the size ratio $X$ from Eq. (4). An observed photocenter oscillation could be due to a small (or very large) satellite orbiting the primary on a large distance, or it could be equally well due to a more moderately sized satellite with a smaller separation from the primary.

A combination of Gaia binary observations with measurements by other technique(s) may provide constraints on the binary parameters. For instance, with $P_{\text{orb}}$ determined from the observations of the photocenter oscillation and a size estimated from observations, e.g., in thermal infrared, the system’s semimajor axis $r$ can be constrained using Kepler’s Third Law, assuming a plausible range of bulk densities. Estimating the size ratio $X$ from Eq. (4) will, however, still remain largely ambiguous.

4.2. Rotational variation of the photocenter

In real asteroid observations, the correction term $C_0$ in Eq. (4) is not zero. Unless the satellite is nearly equal in size to the primary, we can neglect the offset of the photocenter of the secondary’s disk (the last term in Eq. (2)) and the correction term is $C_0 = \frac{1}{I_1} \Delta r_{p1} / D$.

In a given geometry with respect to the Earth and Sun, the primary disk’s photocenter offset vector $\Delta r_{p1}$ generally has a constant and a periodic term due to the rotation of the primary. For a sphere with the Lommel-Seeliger scattering law (see, e.g., Kaasalainen and Torppa, 2001), the projected (the plane-of-sky component of) offset vector is

$$\Delta r_{p1} \approx 0.18D \alpha_\pi,$$

(6)

where $D$ is the diameter of the sphere and $\alpha$ is a phase angle (in radians), see Fig. 5. The offset vector points towards the Sun and it is constant, i.e., its periodic term due to the rotation of the sphere has zero amplitude.

For a non-spherical primary, the periodic component of the offset vector $\Delta r_{p1}$ is non-zero. To estimate a distribution of amplitudes of the periodic component of the offset vector in asteroids that will be observed by Gaia, we run a series of simulations with asteroid shapes represented with ellipsoids as well as with Gaussian random spheres (Muinonen and Lagerros, 1998).

A typical Gaia detected main-belt asteroid will be observed at solar phase about 24°. For this phase angle, a prolate spheroid (the semiaxes $R_a \geq R_b = R_c$) with the Lommel-Seeliger scattering law has a rotational amplitude of the asteroid disk’s photocenter offset of 0.02D and 0.04D for $R_a/R_b = 1.5$ and 2.0, respectively, where $D$ is the diameter of a sphere with equal volume. For the Hapke’s law for macroscopically rough surface (Bowell et al., 1989), we get the rotational amplitude increased by a few percent (see Fig. 6, left panel). For consistency with the amplitude of the photocenter oscillation $A_{pc}$ in Section 3, we define the rotational amplitude as half of a difference between the maximum and the minimum values of the photocenter offset component.

While for the ellipsoid the offset vector $\Delta r_{p1}$ points towards the Sun, for an irregular shape it has a transversal component as well. We computed amplitudes of both components for 1000 Gaussian random spheres rotating around axis with the maximum moment of inertia and observed at phase angle 24°. The results, presented in Fig. 6, show that the photocenters of irregular bodies oscillate with amplitudes typically between 0.02D and 0.08D in the direction towards the Sun. For $D=4\text{ km}$ and a typical geometry of Gaia detected MB asteroid ($a_{\text{b}}=2.5\text{ AU}$; solar elongation $\sim 90^\circ$), this corresponds to a range from 0.05 to 0.20 mas. The transversal component oscillates with amplitudes up to $\sim 0.04D$, corresponding to $\sim 0.10$ mas.

The amplitudes of the rotational variation of the photocenter offset of asteroid disks are similar in magnitude to amplitudes of the photocenter oscillation due to the orbital motion of components of binary systems with small (or very large) secondaries. Thus, we may not be able to resolve, unless perhaps with extensive data and modeling, whether an observed low-amplitude oscillation of the photocenter of an asteroid, with a period of tens or hundreds of hours, is due to a rotational variation of the photocenter in a slowly rotating single asteroid or an oscillation due to the orbital motion of components in a binary system. The simulations presented above suggest that a binary can be uniquely identified with the observations of the photocenter oscillation when

$$A_{pc} \gtrsim 0.08D_{\text{eff}},$$

(7)

where $D_{\text{eff}} = \sqrt{1 + X^2}$ is an effective diameter of the binary system. Using Eq. (4) and assuming circular mutual orbit of the binary, the inequality becomes

$$[(1+X^3)^{-1} - (1+X^2)^{-1}] \gtrsim 0.08 \frac{D_{1} \sqrt{1 + X^2}}{r}.$$

(8)

Using Kepler’s third law and assuming the mass ratio between system’s components $q = X^2$, it is

$$[(1+X^3)^{-1} - (1+X^2)^{-1}] \gtrsim 0.08 \sqrt{1 + X^2} \frac{24\pi}{G \rho (1 + X^2)^{3/2} \rho_{\text{arb}}^2}.$$

(9)
where $G$ is the gravitational constant and $\rho$ is the primary's bulk density. After further processing, we get

$$P_{\text{orb}} \geq 0.212 \frac{(1+X^2)^{3/4}}{\sqrt{\rho(1+X^3)(1+X^3)^{-1}(1+X^2)^{-1}}},$$

with the numerical constant 0.212 computed for $P_{\text{orb}}$ in hours and $\rho$ in g/cm$^3$.

In Fig. 7, we plot the curve representing the minimum $P_{\text{orb}}$ vs size ratio $X$ dependence according to Eq. (10), assuming $\rho = 2$ g/cm$^3$. It is apparent that only systems with small or very large satellites, with $X \leq 0.2$–0.3 or $X \geq 0.8$ for close systems (which we detect in the main belt using the photometric technique), and $X \leq 0.1$ or $X \geq 0.95$ for medium-distance systems with orbital periods on an order of several days (the *Hic sunt leones* range of binary parameters) may not be uniquely distinguishable from slowly rotating single asteroids showing similar photocenter variation due to their rotation.

**5. Conclusions**

Gaia will be the first survey to sample the largely unknown population of medium-distance binary systems among small main-belt asteroids where the present detection techniques (photometric and AO observations) are inefficient. For close systems with orbital periods about 1 day or shorter, which represent a majority in the current sample of asteroid binaries and which were detected with the photometric method of the observations of mutual events between binary components, Gaia will provide only limited data as the close systems show low amplitudes of the photocenter oscillation comparable with or lower than the Gaia's expected astrometric accuracy.

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**References**


Marchis, F., et al., 2008. (3749) Balam. IAUIC 8928.


