PhD Thesis

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Modeling of binary asteroids

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1 Introduction

In recent years a substantial progress has been made in discovery of binary asteroids among all populations in the Solar system, and in characterization of their properties. Various techniques for their detection have been developed: photometry, radars, direct and adaptive optics imaging.

Knowledge of binaries’ rotational and orbital properties is crucial for understanding their origin and evolution. The aim of this work is to describe an inversion method for obtaining some of the parameters from photometry data, and present its limitations.

1.1 Overview of binary asteroids

Binary asteroids have been theorized several times during the twentieth century. André (1901) was the first who speculated that the asteroid Eros was binary on the basis of a similarity of its lightcurve to the lightcurves of β-Lyrae occultation binary stars. Cook (1971) and Weidenschilling (1980) argued that the single body model of Trojan asteroid (624) Hektor, whose observed lightcurve has an amplitude of 1.1 mag., is unstable with respect to a binary fission, and therefore suggested its binary or contact-binary nature. This suggestion was proved to be true from AO observations by Marchis et al. (2006). Although the discovered satellite of (624) is too small to explain large amplitude of the lightcurve (secondary to primary diameter ratio is about 0.05), these observations revealed that the primary is a contact binary itself. Other works on the lightcurve analysis of binary asteroids are discussed in the Section 1.3.

Another indirect evidences for satellites of asteroids were reported during late 1970s, when anomalous observations of star occultations by asteroids were done (Van Flandern et al., 1979). Most of these were visual observations and were probably erroneous. The most reliable indication of an asteroid duplicity from occultation was video observation of (146) Lucina, reported by Arlot et al. (1985), but the proposed satellite with diameter of 6 km wasn’t resolved by space observations (Storrs et al. 1999).

The first direct detection of an asteroid satellite came from the Galileo flyby of the asteroid (243) Ida (Belton and Carlson, 1994), followed by the discovery of a satellite orbiting around (45) Eugenia using adaptive optics on a ground-based telescope (Merline et al. 1999).

Presently the four most effective methods for detecting asteroid satellites are: direct imaging (using space or ground-based telescope), imaging via adaptive optics on ground-based telescopes, radar and photometry, respectively.

Direct and AO imaging is successfully used for revealing binary nature of large or widely separated bodies. To date (July 2007), 51 such objects have been discovered among large and small but wide Main Belt asteroids, among Jupiter Trojans, in Centaur population and among Transneptunian objects using this technique.

The terms primary and secondary refer to larger and smaller component of the system, respectively, in this work.
The first binary revealed by photometry was near-Earth asteroid (NEA) 1994 AW₁ (Pravec and Hahn, 1997). The photometric technique, based on detection of occultation and/or eclipse events in a lightcurve, is restricted for relatively bright and mutually close systems with components size ratios above \( \sim 0.2 - 0.3 \). It also allows in principle a detection of wide systems, but the chance of the events being caught in the lightcurve is decreasing with increasing orbital period of the satellite. Furthermore, the probability of events even to occur decreases with increasing separation of the components (see Pravec et al. 2006). Small binary systems with orbital periods less than \( \sim 40 \text{ hr} \) are found within NEAs, Hungaria group and Inner Main Belt.

The photometric observations are strongly supported by radar, and vice versa. The first radar detection of a binary system was NEA 2000 DP₁₀₇ (Ostro et al. 2000, Margot et al. 2002), subsequently investigated by photometry (Pravec et al. 2006). Including this case the total number of binary systems observed by both methods is 15, thus validating the photometry as a reliable way to detect them. Since a strength of the radar echo is inversely dependent on the fourth power of a distance to the object, the radar technique is limited to close approaches of asteroids to the Earth. On the other hand, the radar imaging during favorable apparitions is able to obtain a very detailed information about shapes, spin states, and a dynamics of mutual orbit of the components. The best studied case using this method is NEA (66391) 1999 KW₄ (Ostro et al. 2006, Scheeres et al. 2006).

The number of confirmed or suspected binary asteroids grows rapidly, Table 1 summarizes current numbers of binary systems and their main properties reported as of April 2007. Parameters of particular systems could be found in references cited in the table.

An important question is what are the real numbers of binary systems in different minor planet populations. A good overview of debiased fraction of binaries provides Noll (2006). A brief summary of his Table 5 is presented in Table 2, complemented by published fractions of binaries in Trojan and Centaur populations.

1.2 Formation, evolution, and stability of binaries

The current complex of binaries fractions in different populations and their properties is assumed to be a steady-state result of their long-term evolution. This evolution encompasses formation, change of orbital characteristics due to tidal (mutual and external) and radiation forces and eventually their destruction. Theories of origin and evolution of binary asteroids should provide testable predictions of their properties in order to compare them with the observations.

Three classes of formation theories are now widely accepted: mutual capture, reaccretion from collisions, and rotational disruption. Richardson and Walsh (2006) provide a good review of this topic so I present here only the most important points with some recent published results.

For Kuiper Belt collision models can be ruled out because they predict too low fraction of binaries. Capture models seems to be more plausible, although the present densities of bodies are too low and encounter speeds too high to allow sufficient number of captures. However, these processes could work in the early solar system and thus a large fraction of Kuiper Belt binaries could be primordial. Re-
Table 1: Numbers of binaries in minor planet populations revealed by different methods and their main properties.

<table>
<thead>
<tr>
<th>Population</th>
<th>Method</th>
<th>detected</th>
<th>references</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main properties</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NEA</td>
<td>radar + phot.</td>
<td>26 (radar 20, phot. 15)</td>
<td>[RW06][Os05][Re06]</td>
</tr>
<tr>
<td></td>
<td>asynchr. (1 sync.), $0.2 &lt; D_s/D_p &lt; 0.5 (0.9)$, $D_p &lt; 5$ km, $2.2 &lt; P_{prim} &lt; 2.8 (14)$ hr, $10 &lt; P_{orb} &lt; 43$ hr</td>
<td>[Be05][Ta05][Be06]</td>
<td></td>
</tr>
<tr>
<td><strong>Mars Crosser</strong></td>
<td>phot.</td>
<td>4</td>
<td>[RW06][Pr07]</td>
</tr>
<tr>
<td></td>
<td>asynchr. (1 sync.), $0.2 &lt; D_s/D_p &lt; 0.3 (0.8)$, $D_p &lt; 7$ km, $2.5 &lt; P_{prim} &lt; 4 (28)$ hr, $10 &lt; P_{orb} &lt; 28$ hr</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Hungaria</strong></td>
<td>phot.</td>
<td>5 (+2 AO)</td>
<td>[RW06][N06]</td>
</tr>
<tr>
<td></td>
<td>asynchr. (1 sync.), $0.2 &lt; D_s/D_p &lt; 0.4 (1)$, $D_p &lt; 5 (10)$ km, $2.5 &lt; P_{prim} &lt; 4 (6.5)$ hr, $14 (6.5) &lt; P_{orb} &lt; 25$ hr</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Main Belt</strong></td>
<td>phot.</td>
<td>15</td>
<td>[RW06][N06][Pr07]</td>
</tr>
<tr>
<td></td>
<td>9 async., 5 sync., $0.2 &lt; D_s/D_p &lt; 0.45 (1)$, $D_p &lt; 6 (10)$ km, $2.2 &lt; P_{prim} &lt; 4 (40)$ hr, $14 &lt; P_{orb} &lt; 40$ hr</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Large Main Belt</strong></td>
<td>AO+DI</td>
<td>10</td>
<td>[RW06]</td>
</tr>
<tr>
<td></td>
<td>asynchr. (1 sync.), $0.03 &lt; D_s/D_p &lt; 0.22 (0.97)$, $D_p &gt; 80$ km, $4 &lt; P_{prim} &lt; 7 (16.5)$ hr, $60 (16.5) &lt; P_{orb} &lt; 115$ hr*</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Small Main Belt</strong></td>
<td>AO+DI</td>
<td>3</td>
<td>[RW06]</td>
</tr>
<tr>
<td></td>
<td>$0.2 &lt; D_s/D_p &lt; 0.4$, $D_p &lt; 6$ km, $1300 &lt; P_{orb} &lt; 2700$ hr</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Trojan</strong></td>
<td>AO</td>
<td>1</td>
<td>[RW06]</td>
</tr>
<tr>
<td></td>
<td>synchr., $D_s/D_p = 0.92$, $D_p = 101$ km, $P_{orb} = 103$ hr</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Centaurs</strong></td>
<td>DI</td>
<td>2</td>
<td>[No06][G06]</td>
</tr>
<tr>
<td></td>
<td>$0.5 &lt; D_s/D_p &lt; 0.9$, $D_p \sim 100 - 200$ km, $200 &lt; P_{orb} &lt; 1000$ hr</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>TNO</strong></td>
<td>AO+DI</td>
<td>27</td>
<td>[RW06][N06][No06b-f]</td>
</tr>
<tr>
<td></td>
<td>$D_s/D_p \sim 1$, $D_p \sim 80$ km, eccentric orbits $19 &lt; P_{orb} &lt; 1000$ hr, $3000 &lt; a &lt; 130 000$ km</td>
<td></td>
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**Notes.** The population of Main Belt binaries is divided into three groups for clarity: the systems detected using photometry are referred to as Main Belt; those detected using adaptive optics and direct imaging are referred as Large and Small Main Belt, distinguishing the systems with primaries greater than 80 km and smaller than 6 km, respectively. The parameters are described in Section 2. Observed extents for asynchronous systems are given in parentheses. Abbreviations: *phot.* = photometry, *AO* = adaptive optics, *DI* = direct imaging. 
* With an exception of (379) Huenna with $P_{orb} = 1939$ hr. 
Table 2: Debiased fraction of binaries in minor planet populations.

<table>
<thead>
<tr>
<th>Population</th>
<th>fraction (%)</th>
</tr>
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<tbody>
<tr>
<td><strong>NEA</strong></td>
<td></td>
</tr>
<tr>
<td>$D_p &gt; 0.3$ km</td>
<td>15 ± 4 (photometry)</td>
</tr>
<tr>
<td>$D_p &gt; 0.3$ km, $2.2 &lt; P_{prim} &lt; 2.8$ hr</td>
<td>66±$^{10}_{12}$ (photometry)</td>
</tr>
<tr>
<td>$D_p &gt; 0.2$ km</td>
<td>16±$^{3}_{5}$ (radar)</td>
</tr>
<tr>
<td><strong>Main Belt</strong></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>∼ 2</td>
</tr>
<tr>
<td>$10 &lt; D_p &lt; 50$ km</td>
<td>∼ 10±7</td>
</tr>
<tr>
<td>Koronis family</td>
<td>22±$^{18}_{9}$</td>
</tr>
<tr>
<td><strong>Trojan</strong></td>
<td></td>
</tr>
<tr>
<td>&lt; 4 $^a$</td>
<td></td>
</tr>
<tr>
<td><strong>Centaur</strong></td>
<td></td>
</tr>
<tr>
<td>ang. separation $&gt; 0.1 - 0.2$ arcsec</td>
<td>13 ±$^{19}_{5}$ $^b$</td>
</tr>
<tr>
<td><strong>TNO</strong></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td>11±$^{5}_{2}$</td>
</tr>
<tr>
<td>large TNOs</td>
<td>75±$^{10}_{30}$</td>
</tr>
</tbody>
</table>

See Noll (2006) and references therein. $^a$ Marchis et al. (2006b), $^b$ Noll et al. (2006).

cently Astakhov et al. (2005) proposed model based on a capture of satellites in chaotic orbits and their subsequent stabilization by successive gravitational interactions with other small objects. Their results match well with the observed moderate mutual eccentricities of binary TNOs and roughly equal masses of their components. Finally, they predicted that a nonnegligible fraction of binaries with larger mass ratios is to be observed.

Durda et al. (2004) performed an extensive study focused on a formation of Main Belt binaries via collisional disruption and reaccretion. Their simulation predict components size ratios comparable to the observed ones, from large ratios of large MB binaries to small systems with small size ratios. The collisional origin of Main Belt binaries is also supported by apparent higher fraction of the systems in asteroidal families. However, Pravec and Harris (2007) pointed out the similarity of small MB systems with a critical total angular momentum (i.e., total angular momentum of a system is close to an angular momentum of an equivalent mass sphere spinning at a disruption limit) and NEA binaries, and proposed a common formation mechanism for both groups.

Since a dynamical lifetime of near-Earth asteroids (∼10 Myr, Gladman et al. 2000) is about an order of magnitude shorter than their collisional lifetime (∼100 Myr), the collisional mechanism is ineffective for this population. An investigation of properties of NEA and small MB binaries revealed that their large components are spinning close to their rotational disruption limit (Pravec et al., 2006, 2007) and that the systems have the total angular momentum content close to the critical value.

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This indicates that these objects are formed through the rotational disruption. There are two ways for asteroids to reach the spin barrier: planetary flybys and radiative spin-up caused by YORP effect (see Bottke et al. 2006 and references therein). The former was revisited by Walsh and Richardson (2007). They found that the tidal disruption should account for 1-2% of NEA being binaries, i.e., approximately 7-13% of observed NEA binaries can be formed by this mechanism. Because there is no sufficiently large and/or dense object in the Main Belt, tidal disruption is not considered an important mechanism for forming binaries in that population. A powerful way for small asteroids to reach the spin barrier within their dynamical lifetime is the latter mechanism – the YORP effect. The actual physics of binary formation from critically rotating body haven’t been modeled yet. But, as showed by Scheeres et al. (2006), a material on the surfaces of rapidly spinning asteroids can be subjected to almost weightless environment, and thus some sort of successive mass shedding or “landslide” can occur.

After the satellite is formed, the system is subjected to many influences. “Classical” effects caused by mutual tidal forces were summarized by Weidenschilling et al. (1989). These include a synchronization of the satellite rotational period with the orbital period, an increasing of the semimajor axis of the mutual orbit and a decreasing of its eccentricity. Margot et al. (2002) showed that the tidal despinning of a satellite of binary NEA 2000 DP$_{107}$ is two orders of magnitude shorter than the age of the system (strictly speaking, than the time scale for the secondary’s orbit evolve to its present size, if an assumption that the secondary was close to the primary at the time of formation is used). Since the properties of this binary are similar to the other binary NEAs and photometrically detected small MB binaries, it is expected that its satellites would be found in the spin-locked state, which is confirmed by the observations. The timescales needed to reach a fully synchronous state (i.e., both rotational periods are equal to the orbital period) are much longer and depends strongly on the mass ratio of the components. Thus, only the systems with large satellites are observed in this state (e.g., NEA 69230 Hermes and MB binary 90 Antiope).

The eccentricity of the mutual orbit evolves quickly to low values for small satellites and small separations of components. Walsh and Richardson (2006) showed that its timescales for most of the observed binary NEAs are shorter than 10 Myr, consistently with their observed eccentricities < 0.1. The exceptional case is 1998 ST$_{27}$ with the timescale larger than 100 Myr and the observed eccentricity > 0.3.

The eccentricity, as well as separation of components, of binary NEAs can be also raised by tidal encounters with Earth or Venus (Bottke and Melosh 1996). Since the effect of eccentricity increase is dependent on a separation of the components in this case, it could represent another suggestion that wide-separated NEA binaries may have high eccentricities, but no statistical analysis has been done so far.

Since most of the observed satellites orbit within 0.1 of their primaries’ Hill sphere radii$^2$ (Noll 2006), they are not significantly affected by solar tides. Chauvineau and Mignard (1990) shown that Main Belt binary asteroids are dynamically stable

$^2$The radius of the Hill sphere is defined here as $R_{Hill} = a_{hel}/(m_p/3M_\odot)^{1/3}$, where $a_{hel}$ is a semimajor axis of a system’s heliocentric orbit, $m_p$ is the mass of the primary and $M_\odot$ is the mass of the Sun.
against solar and jovian tides over the age of the solar system if the separation of components is smaller than a few tens of primary radii. But the observed upper limit on the separation of components, observed in all populations where binaries were detected, suggest that wider systems have been subjected to tidal disruption. Indeed, small solar perturbations occur – Scheeres et al. (2006) suggested that the mutual orbit of binary NEA (66391) 1999 KW$_4$ is excited during perihelion passages.

However, survival of binaries is not guaranteed against close approaches to other solar system bodies. Walsh and Richardson (2007) showed that a mean lifetime of NEA binaries including effects of planetary tides is only 1-2 Myr.

Petit and Mousis (2004) showed that wider TNO binaries like 1998 WW$_{31}$ and 2001 QW$_{322}$ have lifetime against a collisional unbinding or a gravitational perturbation by another TNO of order 1-2 Gyr, which implies that these system had to be between 7 and 50 times more numerous 4 Gyr ago, if they are primordial.

The influence of the thermal forces is increasing towards the Sun and thus may play an important role in the evolution of binary NEAs. Çuk and Burns (2005) modeled the effect of YORP on components of binary NEAs and showed that changes in separations, eccentricities and inclinations of the mutual orbits have typical timescales as short as $10^5$ yr, and thus dominating tidal evolution.

1.3 Previous work on modeling of lightcurves of binaries

In fact, to the author’s knowledge, Zappalà et al. (1980) were the first to make a synthetic lightcurve of binary asteroid. Their model encompassed eclipses and mutual shadowing effects at different phase angles. Although they did not model real data, they accidentally chose parameters and phase angle similar to the observed properties and geometry of binary NEAs, and thus synthetic lightcurves in their Fig. 1 strongly resemble the long-period components of lightcurves shown on Fig. 12 in Section 3.1 of this thesis.

Another early theoretical work by Leone et al. (1984) was devoted to a lightcurve morphology of a synchronous pair of homogenous bodies in hydrostatic equilibrium, modeled by a couple of Roche ellipsoids.$^3$ They investigated lightcurve features, such as lengths of partial and total events, total amplitude and an amplitude without events, of the system observed equatorially at opposition, as a function of mass ratio of the components.

Applying the above-mentioned technique, Cellino et al. (1985), tried to fit large-amplitude lightcurves of some Main Belt asteroids suspected of being binary. They obtained acceptable agreement between synthetic and observed lightcurves, but none of the asteroids have been shown to be real separated binaries by other methods yet (Tanga et al. 2003, Marchis et al. 2006), except for (624) Hektor (see Section 1.1).

After the discovery of the asynchronous systems with doubly periodic lightcurves in the 1990s, the first successful attempts to model their lightcurves were made by Pravec et al. (2000) and Mottola and Lohullra (2000) in the case of NEA 1996 FG$_3$. Although the used data and numerical implementation of their models differ from

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$^3$In fact this is a simplification of the problem, because Roche ellipsoid is an equilibrium shape of body orbiting a mass point, and the deformation caused by the deviations of the primary’s gravitational field from the spherical symmetry is neglected.
each other, they obtained similar results, providing an independent verification. In the former work, trial coordinates of a mutual orbit’s pole were used, but the pole location agree within 10 degrees with a retrograde solution of the latter work, where a full-grid search of the orbital pole was carried out. Another outcome of the above-mentioned works was that the synthetic lightcurves of binaries are only weakly sensitive to the choice of a light-scattering model (see also Section 4.2). While Pravec et al. used a very simple assumption that a total measured flux is proportional to a sum of projected non-occulted and non-shadowed areas of both components, Mottola and Lahulla used Hapke scattering function (Hapke 1984).

Pravec et al. (2002b) presented another simple model of long-period component of lightcurve obtained for asynchronous NEA 1999 HF₁.

An interesting technique of solving for the mutual orbit’s pole of (90) Antiope, a synchronous equal-sized binary Main Belt asteroid, was developed by Michalowski et al. (2002). The asteroid was observed during multiple apparitions with differing amplitudes and durations of mutual events in the lightcurve. Using these quantities independently and simple geometrical considerations they obtained qualitatively similar pole solutions.

While the asynchronous systems are clearly distinguished by the two periods in their lightcurves (see Section 2.1.1), the synchronous binaries could be photometrically detected just on the basis of the shape of the lightcurve. Dürech and Kaasalainen (2003) (further referred as [DK03]) synthesized lightcurves for several real and artificial highly nonconvex shapes and synchronous binary systems, and determined a minimal solar phase angle $\omega_{\text{min}}$ at which the lightcurves cannot be fitted by a convex model. They showed that this angle is clearly dependent on the so-called nonconvexity measure defined as $V_{nc} = 1 - V/Vch$, where $V$ is a total volume of the asteroid and $V_{ch}$ is the volume of its convex hull.

The results of [DK03] can be used for four Main Belt binary candidates with single-period lightcurves reported by Behrend et al. (2006). Values of maximum phase angles at which these asteroids were observed fall within a range of 19°–29°, while their nonconvexity measures calculated from reported parameters of the systems (sizes and distances of components) are between 0.45 and 0.59. Extrapolating from Fig. 7 of [DK03], the maximum phase angles are just near the minimal values $\omega_{\text{min}}$ needed for detection of nonconvexities. Nevertheless, the observed lightcurves (see Fig. 1) support the hypothesis of the binarity because of their sharp “shoulder-like” shape.

On the contrary, Takahashi and Ip (2004) modeled the single-period lightcurve of a Kuiper Belt object with large amplitude as Roche binary system. Since the observations were made at small solar phase angle (below 1°), the convex shape model would fit the lightcurves as well.

Extensive work on the topic was performed by Pravec et al. (2006), although the main scope of this paper was intended to derive basic properties of the NEA binary population as a whole. Of the seventeen asynchronous binary NEAs whose data were analyzed, four (one in two apparitions) lightcurve datasets were inverted in a least-square sense to provide basic parameters of the systems. All these four asteroids are revisited in this thesis as well.

Lacerda and Jewitt (2007) found that two of five icy bodies they studied have
Figure 1: Phased lightcurve of synchronous binary asteroid (854) Frostia. Reprinted from Behrend et al. (2006) with kind permission of R. Behrend.

lightcurves well described by Roche contact binary model. They also showed that parameters of best fit solutions obtained for these two binaries aren’t strongly affected by a choice of scattering function (Lommel-Seeliger law for low albedo lunar-type and Lambert law for high albedo icy-type surfaces).
2 Methods

This section focuses on analysis of photometric data of binary asteroid and their inversion. In the first three subsections, semianalytic methods for deriving some basic parameters from the lightcurve are described. The fourth and the fifth subsections are devoted to direct and inverse problem, respectively.

2.1 Principles of photometric detection of binaries

2.1.1 Doubly periodic lightcurves of asteroids

Photometric data of a single asteroid rotating around axis with the maximal moment of inertia (the so-called principal axis rotator), reduced to unit distances from the Earth and the Sun and to given phase angle, form a single periodic function of magnitude or intensity versus time, providing that the time interval is short enough that a changing aspect would not affect the lightcurve shape significantly. The period of the lightcurve could be estimated using the Fourier series method (see Harris et. al. 1989, Pravec et al. 1996).

If the observed data show systematic deviations from this periodic function, which cannot be attributed to the changing geometry, one or more additional periods have to be searched in the lightcurve. The multiple periodic lightcurves can be produced by two types of bodies:

1. a non-principal axis (NPA) rotation of a single body,

2. a system of two bodies with different rotational periods (i.e., asynchronous binary asteroid).

Since the NPA state is described in detail elsewhere (e.g., see Pravec et al. 2005), only main differences of the two cases are presented here.

If two periods are present in the lightcurve, it could be represented as a sum of Fourier series in the following general form (see Pravec et al. 2005)

\[
F(t) = C_0 + \sum_{j=1}^{m} \left[ C_{j0} \cos \frac{2\pi j}{P_1} (t - t_0) + S_{j0} \sin \frac{2\pi j}{P_1} (t - t_0) \right]
+ \sum_{k=1}^{m} \sum_{j=-m}^{m} \left[ C_{jk} \cos \left( \frac{2\pi j}{P_1} + \frac{2\pi k}{P_2} \right) (t - t_0) + S_{jk} \sin \left( \frac{2\pi j}{P_1} + \frac{2\pi k}{P_2} \right) (t - t_0) \right],
\]

where \(F(t)\) is the total reduced light flux at time \(t\), \(t_0\) is a zero-point time, \(C_0\) is a mean reduced light flux, \(C_{jk}\) and \(S_{jk}\) are the Fourier coefficients of the corresponding linear combinations of the two periods \(P_1\) and \(P_2\), and \(m\) is the highest significant order.

A signal (normalized to the mean reduced light flux) of the combination of two frequencies \(\left( \frac{j}{P_1} + \frac{k}{P_2} \right)\) is expressed using coefficients from the above expansion as

\[
A_{jk}^{\text{norm}} = \sqrt{C_{jk}^2 + S_{jk}^2}/C_0.
\]
Figure 2: Lightcurve of non-principal axis rotating asteroid 2002 TD₉₀ (observed data and synthetic lightcurve). Reprinted from Pravec et al. 2005

In the first case of NPA rotator (see Fig. 2) the lightcurve contains significant signal in the combination of both frequencies, i.e., at least some of the coefficients \( C_{jk} \) and \( S_{jk} \) are nonzero for \( j \neq 0 \) and \( k > 0 \).

In the second possibility, the components of the binary system both produce its own rotational lightcurve and their fluxes add linearly. Thus, the coefficients \( C_{jk}, S_{jk} = 0 \) for \( j \neq 0 \) and \( k > 0 \), and Eq. (1) has the form

\[
F(t) = C_0 + \sum_{j=1}^{m} \left[ C_{j0} \cos \frac{2\pi j}{P_1} (t - t_0) + S_{j0} \sin \frac{2\pi j}{P_1} (t - t_0) \right] \\
+ \sum_{k=1}^{m} \left[ C_{0k} \cos \frac{2\pi k}{P_2} (t - t_0) + S_{0k} \sin \frac{2\pi k}{P_2} (t - t_0) \right].
\]  

(3)

If the system is asynchronous, \( P_1 \neq P_2 \) and both lightcurve components are resolved in a combined lightcurve. When the Earth or the Sun are close to the orbital plane of the system, occultation or eclipses, respectively, are observed in the lightcurve (an example of such lightcurve is shown in Fig. 3). These phenomena are collectively called mutual events, and the terms “primary event” and “secondary event” are used according to which body is being occulted/eclipsed. If the rotation of the secondary component is nonsynchronous with its orbital motion and the mutual events are present, all three periods (two rotational and one orbital) could be resolved.

### 2.1.2 Synchronous binaries

As already noted in Section 1.3, fully synchronous binaries cannot be detected by the period analysis, since their lightcurve is monoperiodic. If the mutual events do not occur, the photometric data are indistinguishable from lightcurve of single asteroid and the binary nature of asteroid is definitely missed. If the size ratio of components is sufficiently small and the geometry is favorable for the events to

---

4The lightcurve of NPA rotator is composed of two periods – a period of rotation around one of the principal axes, and a period of precession of the axis around asteroid’s angular momentum vector.
be present in the lightcurve, they could be recognized by their sharp borders (see Fig. 1 or Michalowski et al., 2002). Fast evolution of the event shapes and depths with changing geometry could also suggest the binary nature. It should be noted, however, that this sort of lightcurve could also be produced by extremely nonconvex single bodies (e.g. dog-bone shape like (216) Kleopatra).

2.2 Radius vectors of secondary body in apparent contacts

In some of the following sections I describe algorithms enabling the lightcurve inversion to be very fast. The algorithms need as input radius vectors of the secondary with respect to the primary in instants of time when certain mutual events begin or end, defined in an inertial reference frame.

These instants are called contacts\(^5\) and the notation used is following: the first and the fourth contacts denote the beginning and the end of the partial event, while the second and the third contacts denote the beginning and the end of the total event, respectively. Since the radius vectors of the secondary at the contacts are used more than once in the following sections, their derivation deserves separate section here.

It is assumed in the derivation that the geometry with respect to the Earth and the Sun, and the pole and the semi-major axis of the mutual orbit are given, as well

\(^5\)These contacts are apparent only and do not have any relation to the "contact binaries".
as the size ratio of the components, and that their shapes are spherical.

Another assumption is that a distance between the asteroid and the observer is known, and therefore all observations could be corrected for light-travel time and calculations are done in the asteroid’s time frame.

### 2.2.1 Circular orbit

Define a unit vector $\mathbf{c}$, heading from primary’s to secondary’s center at the time of some of the contacts for an eclipse of the primary. Let us assume the shapes of both components are spherical, with radii $R_p$ and $R_s$ for primary and secondary, respectively, and the secondary orbiting around the primary on a circular orbit with a radius $r$. Denote $\theta_1$ an angle between $\mathbf{c}$ for the first (or the fourth) contact and the unit vector heading in the direction to the Sun, $\mathbf{s}$.

Similarly, $\theta_2$ is the same angle for the second and the third contact (see Fig. 4). Normalizing $R_p$ to unity, these angles are defined by equations

$$
\sin \theta_1 = \frac{1 + R_s}{r}, \\
\sin \theta_2 = \frac{1 - R_s}{r}.
$$

The vector $\mathbf{c}$ for the first and the fourth contact is then defined by a set of equations

$$
\mathbf{c} \cdot \mathbf{s} = \cos \theta_1, \\
\mathbf{c} \cdot \mathbf{n} = 0,
$$

where $\mathbf{n}$ is a unit vector perpendicular to the secondary’s orbit.

The above-defined angles allow us also to easily write conditions for the eclipse events to occur. An angle between $\mathbf{s}$ and $\mathbf{n}$, $\beta_s$, must satisfy

$$
|\beta_s - 90^\circ| < \theta_1
$$

for at least partial events, and

$$
|\beta_s - 90^\circ| < \theta_2
$$

for total events. Since $\beta_s \in (0, \pi)$, the above conditions could be rewritten to more timesaving expressions

$$
|\cos \beta_s| < \sin \theta_i, \ (i = 1, 2), \text{ where } \cos \beta_s = \mathbf{s} \cdot \mathbf{n}.
$$

Solving equations (6), (7) and $|\mathbf{c}| = 1$ (see Appendix 6.2), we get two solutions for $\mathbf{c}$:

$$
\mathbf{c} = \frac{\alpha \mathbf{a} + \mathbf{b}}{a^2},
$$

\(^{6}\text{Note that the vector } \mathbf{s} \text{ do not lie in the secondary’s orbital plane, in general.}\)
where

\[
\mathbf{a} = \mathbf{n} \times \mathbf{s}, \quad (12)
\]
\[
\mathbf{b} = \mathbf{a} \times (\cos \delta_1 \mathbf{n}), \quad (13)
\]

and

\[
\alpha = \pm \sqrt{a^2 - b^2/a^2}. \quad (14)
\]

We are denoting these two solutions as \( \mathbf{c}_{pe1} \) and \( \mathbf{c}_{pe4} \) for the first and the fourth contact of the primary eclipse, respectively.

Positions of the secondary at the second \( \mathbf{c}_{pe2} \) and the third \( \mathbf{c}_{pe3} \) contacts could be found in the same way, using an angle \( \delta_2 \) instead of \( \delta_1 \).

Substituting \( \mathbf{s} \) with \(-\mathbf{s}\) in Eq. (12), we obtain positions of the secondary in four contacts of eclipse of secondary, \( \mathbf{c}_{se1} \) to \( \mathbf{c}_{se4} \). Finally, substituting \( \mathbf{s} \) with the vectors heading in the direction to the Earth, \( \mathbf{h} \), and in the opposite sense, \(-\mathbf{h}\), we obtain contacts of secondary’s transit in front of the primary, \( \mathbf{c}_{p01} \) to \( \mathbf{c}_{p04} \), and of secondary occultation, \( \mathbf{c}_{sol} \) to \( \mathbf{c}_{sol4} \), respectively.

We are assuming in the previous derivations that the geometry is constant during the whole event (i.e. the vectors \( \mathbf{s} \) and \( \mathbf{h} \) are fixed). If the geometry is changing rapidly, the vectors \( \mathbf{c} \) have to be searched iteratively.

### 2.2.2 Elliptic orbit

In the case of an elliptic orbit, the distance between the centers of the primary and the secondary is not constant over time. The angles \( \delta_1 \) and \( \delta_2 \) are then defined by

\[
\sin \delta_1 = \frac{1 + R_s}{r}, \quad (15)
\]
\[
\sin \delta_2 = \frac{1 - R_s}{r}, \quad (16)
\]

where

\[
r = \frac{a(1 - \epsilon^2)}{1 + \epsilon \cos v}, \quad (17)
\]

\( a \) is a semimajor axis of the mutual orbit, \( \epsilon \) is its eccentricity and \( v \) is a true anomaly of the secondary at the contact. As in the previous paragraphs, \( R_p \) is normalized to unity.

The necessary conditions for events to occur are

\[
\gamma < \delta_i^{max}, \quad (18)
\]

while the sufficient conditions are

\[
\gamma < \delta_i^{min}, \quad (19)
\]

\((i = 1 \text{ for at least partial events and } i = 2 \text{ for total events})\) where

\[
\gamma = |\beta_s - 90^\circ|, \quad (20)
\]
\[
\sin \delta_i^{max} = \frac{1 + R_s}{a(1 - \epsilon)} \quad (21)
\]
Figure 4: Vectors and angles used in the derivation of the eclipse contacts (only the case of circular orbit is shown here): $\mathbf{c}_{\text{se}1}$ to $\mathbf{c}_{\text{se}4}$ are unit vectors heading from the primary’s to the secondary’s center at the first to the fourth contact during secondary’s eclipse, $\mathbf{n}$ is the normal to the mutual orbit plane, $\mathbf{s}$ is a vector heading in the direction to the Sun, $\delta_1$ is an angle between $\mathbf{c}_{\text{se}1}$ and $-\mathbf{s}$ ($\mathbf{c}_{\text{se}4}$ and $-\mathbf{s}$), $\delta_2$ is an angle between $\mathbf{c}_{\text{se}2}$ and $-\mathbf{s}$ ($\mathbf{c}_{\text{se}3}$ and $-\mathbf{s}$), and $\beta_s$ is an angle between $\mathbf{n}$ and $\mathbf{s}$. $L_{\text{pc}1}$ and $L_{\text{sc}1}$ are arguments of mean lengths of the secondary in the first contacts of secondary’s shadow with the primary and the primary’s shadow with the secondary, respectively. For the circular orbit, the angles $L_{\text{pc}1}$ and $L_{\text{sc}1}$ are equal to oriented angles between the ascending node and the corresponding vectors $\mathbf{c}$. Similarly, the angles $L_{\text{pc}4}, L_{\text{sc}2}, \ldots$ could be defined for vectors $\mathbf{c}_{\text{pc}4}, \mathbf{c}_{\text{sc}2}$, etc. The purpose of introducing these angles is given in Section 2.3.1. The shadows of the primary and the secondary are indicated as gray area.
\[
\sin \delta_1^{\text{min}} = \frac{1 + R_s}{a(1 + e)}, \quad (22)
\]
\[
\sin \delta_2^{\text{max}} = \frac{1 - R_s}{a(1 - e)}, \quad (23)
\]
\[
\sin \delta_2^{\text{min}} = \frac{1 - R_s}{a(1 + e)}. \quad (24)
\]

The vector \( \mathbf{c} \) for the first, the fourth \((i = 1)\), the second and the third \((i = 2)\) contact is defined by a set of equations

\[
\mathbf{c} \cdot \mathbf{s} = \cos \delta_i, \quad (25)
\]
\[
\mathbf{c} \cdot \mathbf{n} = 0, \quad (26)
\]
\[
|\mathbf{c}| = 1, \quad (27)
\]
\[
\mathbf{c} \cdot \mathbf{p} = \cos v, \quad (28)
\]

where \( \mathbf{p} \) is a unit vector heading to the mutual orbit’s pericenter.

The vector \( \mathbf{c} \) and the true anomaly \( v \) are found simultaneously by minimization of a function

\[
f(v) = (\mathbf{c} \cdot \mathbf{s} - \cos \delta_i)^2, \quad (29)
\]

where optimization parameter is \( v \). The searching interval for \( v \) could be reduced to \( (v_{\text{min}}, v_{\text{max}}) \), where \( v_{\text{min}} = \min(v_1, v_2) \) and \( v_{\text{max}} = \max(v_1, v_2) \). The boundary values \( v_1, v_2 \) are equal to mean anomalies \( M_1, M_2 \) from the circular approximation of orbit, where \( r = r_{\text{max}} \) for \( M_1 \) and \( r = r_{\text{min}} \) for \( M_2 \). The values of \( r_{\text{min}} \) and \( r_{\text{max}} \) are obtained using following rules:

\[
r_{\text{min}} = a(1 - e),
\]

if \( \gamma < \delta_i^{\text{min}} \):
\[
r_{\text{max}} = a(1 + e)
\]

else:
\[
r_{\text{max}} = \frac{1 + R_s}{\sin \gamma} \quad \text{for partial events} \ (i = 1),
\]
\[
r_{\text{max}} = \frac{1 - R_s}{\sin \gamma} \quad \text{for total events} \ (i = 2).
\]

The function \( f \) is evaluated in a following scheme. For given \( v \):

1. compute \( \delta_i \) from eqs. (15, 16) and (17),
2. compute \( \mathbf{c} \) from eqs. (26), (27) and (28) (in the same way as the vector \( \mathbf{c} \) is computed from \( \mathbf{s}, \mathbf{n} \) and \( \cos \delta_i \) in the circular case).
3. compute \( f(v) = (\mathbf{c} \cdot \mathbf{s} - \cos \delta_i)^2 \).

If the minimum \( \min(f(v)) > 0 \), then the solution for the given combination of parameters does not exist, i.e., the certain event cannot occur in the lightcurve.
2.3 “Preprocessing” of lightcurve with mutual events

The advantage of the two linear additive components in the lightcurve of the asynchronous binary is that they could be separated by fitting the doubly period Fourier series (Eq. 3). Since the mutual events are superimposed onto the sum of rotational lightcurves expressed in Eq. (3), this fitting has to be done using only data points outside the events. One or the other component could be subsequently subtracted from the lightcurve data to obtain the short- or long-period component of the lightcurve. This procedure is described in Pravec et al. (2006) in detail.

The following notation is used in the rest of the thesis: $P_1$ and $P_2$ denote the periods of the short- and long-period components fitted to the observed data, which are believed to be synodic rotational periods of the primary and the secondary, respectively. The period of recurrence of the mutual events is interpreted as synodic orbital period of the system and denoted as $P_{orb}^{syn}$. All symbols used in the text and their explanation are summarized in Appendix (Section 6.1).

The presence of the mutual events enables to estimate some other parameters from their properties. If the secondary is totally occulted or eclipsed, a plateau of constant brightness occur in the secondary event (see, e.g., Fig. 20). For same albedos and phase effects of the two bodies, the depth of this plateau is related with the ratio of mean projected diameters as

$$\Delta m = 2.5 \log \left( \frac{C_0}{C_1} \right) = 2.5 \log \left[ 1 + \left( \frac{D_s}{D_p} \right)^2 \right],$$

(30)

where $C_0$ is defined by Eq. 3, $C_1$ is mean reduced light flux of short-period component, $D_s$ and $D_p$ are mean projected diameters of secondary and primary, respectively.

The assumption that the two components have a same albedo was checked by Pravec et al. (2006) for systems with best obtained data and was found to be held to within 20%. Data of systems modeled in this work are consistent with the assumption as well.

2.3.1 Events contacts method for estimating $P_{orb}^{sid}$ and $L_0$

Since the period of recurrence of the mutual events includes the synodic effect due to a motion of the system with respect to the Earth and the Sun, this period is synodic. For a given pole and a radius of the mutual orbit (assumed or sampled on some grid), however, sidereal orbital period $P_{orb}^{sid}$ can be derived from time instants of the contacts defined in Section 2.2.

First, the contacts have to be identified in the long-period component of the lightcurve. This is done visually on the basis of rapid decrease or increase of the brightness. Some contacts, especially the second and the third contacts of primary events, do not produce this rapid change in brightness due to a mutual interference of occultations and eclipses. These contacts, which cannot be identified visually, are omitted in analysis.

---

\textsuperscript{7}It should be noted that for most of the observed cases $P_2$ and $P_{orb}^{syn}$ are equal or indistinguishable.
If the plateau is observed in the lightcurve$^8$, the contacts could be divided into six types, designated as \textit{PPB, PPE, SPB, STB, STE} and \textit{SPE} which refer to beginning of primary partial event, end of primary partial event, beginning of secondary partial event, beginning of secondary total event, etc., respectively. An example of the contacts identified in the observed data is shown in Fig. 5. Note that the contacts and their abbreviations defined above are different from those in Fig. 10, in which the pole of the mutual orbit is known and the nature of the events (i.e., whether they are occultations or eclipses) is resolved already.

![Figure 5: Example of contacts identified visually in the observed data of NEA 1998 RO$_1$. The abbreviations used are defined in Section 2.3.1.](image)

All identified contacts in the lightcurve are sorted by the time of their occurrence, $T_i$ ($i = 1..N$, where $N$ is the number of contacts). For each $T_i$ the geometry (vectors $\mathbf{s}$ and $\mathbf{h}$) is known from the heliocentric orbit, and for given components size ratio, mutual orbit pole and radius the vectors $\mathbf{c}$ are computed using circular approximation described in Section 2.2.1. The use of angles $\delta_1$ or $\delta_2$ in Eq. 13 depends on whether the contact defines a margin of partial or total event, respectively, while the primary or the secondary type of event determines the sign of vector $\mathbf{s}$ (or $\mathbf{h}$) in Eq. 12. Whether the event is an occultation or an eclipse is unknown, and both possibilities have to be examined by using vectors $\mathbf{s}$ and $\mathbf{h}$ in Eq. 12, respectively. Thus, none, one or two solution of vector $\mathbf{c}$ can be found for each contact with epoch $T_i$. The whole set of these solutions can be expressed as

$$C = \{C_i\}, \quad i = 1..N,$$

where $C_i$ is a set of $m_i$ vectors $\mathbf{c}_{i,j}$, $j \in [1, m_i]$, $m_i \in [0, 2]$. If $m_i = 0$ then $C_i = \emptyset$.

If any of $m_i$ has zero value, then the tested combination of components size ratio, mutual orbit pole and radius is rejected for further analysis.

$^8$In other case only four types of contacts could be identified, and their assignment to primary and secondary events have to be treated as two independent possibilities.

$^9$E.g., if $N = 5$, $m_1 = 2$, $m_2 = 1$, $m_3 = 1$, $m_4 = 0$ and $m_5 = 2$, then the whole set $C$ contains vectors $\mathbf{c}_{1,1}; \mathbf{c}_{1,2}; \mathbf{c}_{2,1}; \mathbf{c}_{3,1}; \mathbf{c}_{5,1}; \mathbf{c}_{5,2}$.
Once we have vectors \( c_{i,j_i} \), we can calculate their corresponding arguments of mean lengths \( L_{i,j_i} \) as oriented angles between the ascending node and \( c_{i,j_i} \).\(^{10}\) These arguments are then converted to time-increasing (not restricted to interval \([0,2\pi)\) only) values defined as

\[
L'_{i,j_i} = L_{i,j_i} + k2\pi,
\]

where \( k \) is the number of cycles\(^{11}\) that passed from \( T_i \).

If the time instants of contacts, \( T_i \), were established from lightcurve with absolute precision, the mutual orbit was circular, and only one solution existed for each \( T_i \) (i.e., \( m_i = 1 \)), then the pairs of \([L'_{i,1},T_i]\) would lie on a straight line defined by

\[
L' = nT + L_0,
\]

where \( n \) is an angular velocity, or mean motion, of the secondary, and \( L_0 \) is its argument of mean length for \( T = 0 \). Since this is obviously not true in a real case, every combinations of \( j_i \) in \( L'_{i,j_i} \) are used to construct series of the pairs of \([L'_{i,1},T_i]\). Parameters \( n \) and \( L_0 \) are then fitted by least-squares fit of Eq. (33) for each of these series, and their values for a series with the smallest dispersion are taken as the most probable values. The synodic orbital period of the secondary is then expressed simply as

\[
P_{\text{orb}} = \frac{2\pi}{n}
\]

The above-described algorithm can be applied for all values of mutual orbit pole and radius sampled on some grid. Since it is based on fitting of just a few time instants identified in the lightcurve, it allows the construction of a map of initial values of the parameters to be time-effective. Examples of such maps are shown in Fig. 6.

### 2.4 Lightcurve computation – the direct problem

#### 2.4.1 Shape of bodies

The shapes of both components can be represented by spheres, ellipsoids, or other arbitrary shapes approximated by polyhedra with triangular facets. Their vertices are distributed so that the facets have roughly the same area in the case of spherical shape. If other than spherical shape is used, then the vertices are generated on sphere and then projected onto its surface (see Fig. 7).

Usually, 1016 and 252 are sufficient numbers of facets used for the primary and the secondary, respectively. The dependence of computational noise on the number of facets is discussed in Section 4.3 in more detail.

When modeling the long-period component of the lightcurve, an axially symmetric shape should be used for the primary in order to smooth away its rotational component. Besides generating the short-period component of the lightcurve, the actual shape of the primary also influences exact timings of the mutual events. Thus, the

---

\(^{10}\) The argument of mean length is defined as \( L = \omega + M \), where \( \omega \) is an argument of pericenter and \( M \) is a mean anomaly.

\(^{11}\) This number is calculated using the first guess of orbital period equal to \( P_{\text{orb}}^{\text{sym}} \).
Figure 6: Example of maps of parameters precomputed from contacts identified in the lightcurve of (65803) Didymos for various direction of the mutual orbit’s normal (defined by ecliptic coordinates $\lambda_p$ and $\beta_p$). The precomputed parameters are (from top to bottom) $a/R_p$($=2a/D_p$), $P_{\text{sid}}$, $P_{\text{orb}}$, and $L_0$. Values of the parameters are coded by gray shades from black to white.
right way to simulate the long-period component is to construct complete, multipleperiod lightcurve using nonsymmetric shape for both bodies, and then subtract the short-period component from the lightcurve as described in Pravec et al. (2006).

However, since the data of binary asteroids modeled in this work do not cover sufficient range of geometries necessary for determining their accurate shape (see Table 3), the above procedure is not used and the shapes of the bodies obey the following approximations:

The shape of the primary is assumed to be an oblate spheroid, with a spin axis taken to be normal to the orbital plane of the secondary for simplicity. Detailed modeling of radar data obtained for binary system 1999 KW₄ (see Östro et al., 2006) showed that the primary’s rotational pole and the mutual orbit normal have nominal separation of 3.2°, confirming the assumption for this case. The flattening of the primary shape is expressed as its equatorial to polar semiaxes ratio, \(a_p/c_p\).

In order to simulate an amplitude of the long-period component of the lightcurve, the shape of the secondary is taken as a prolate spheroid synchronously rotating so that its long axis is aligned with the centers of the two bodies at the pericenter. The elongation of the secondary shape is expressed as its equatorial (the largest) to polar (the shortest) semiaxes ratio, \(a_s/c_s\). A size ratio of the components is expressed as a ratio of their largest semiaxes, \(a_s/a_p\).

Since Eq. 30 gives only an estimation of \(D_s/D_p\) from the drop in the lightcurve, a conversion between parameters used in the model and \(D_s/D_p\) is needed. I therefore define \(D_s/D_p\) as a ratio of rotation-averaged projected areas of the components’ equator-on silhouettes.

While the rotating primary seen from the equator has a constant projected area equal to \(a_p c_p\), the secondary’s projected area vary in time, and therefore it is approximated with it’s rotation-averaged value defined as \(\sqrt{A_x C_x C_s}\). \(D_s/D_p\) is then expressed as

\[
D_s/D_p = \sqrt{\frac{\sqrt{A_x C_x C_s}}{a_p c_p}},
\]

which may be rewritten using the ratios defined in this section as

\[
D_s/D_p = \sqrt{\frac{a_p/c_p}{(A_x/c_s)^{3/2}(a_s/a_p)^2}}.
\]

2.4.2 Properties of mutual orbit

The system is dynamically treated as two mass points orbiting each other without external forces, i.e., the orbit is Keplerian and its pole is fixed in the inertial reference frame.\(^\text{12}\)

The orientation and the shape of mutual orbit are described by five parameters: pole coordinates in ecliptic frame \(\lambda_p, \beta_p\) (Epoch J2000), the semimajor axis-to-primary equatorial semiaxis ratio \(a/a_p\), the eccentricity \(e\), and the argument of

\(^\text{12}\)In fact, a correction for an oblateness of the primary is taken into account in determining a bulk density (Section 2.5.1).
Figure 7: Polyhedra with 572 triangular facets used to approximation of sphere and ellipsoid with $a:b:c$ semiaxes ratio equal to 6:5:4. See text for details.

pericenter $\omega$. Since a bulk density of the bodies is not known, $a/A_p$ and the sidereal period $P_{s}^{\text{sid}}$ have to be searched as independent variables.

The advantage of using $a/A_p$ (instead of, e.g., $a/C_p$) is that for occultation/eclipse events occurring near the primary’s equator, as is the case for most of observed binaries, the length of events is directly proportional to this parameter.

The classical mean anomaly of the secondary for a given epoch $T_0$ is replaced with the argument of mean length $L_0$ (i.e., the angular distance from the ascending node). The reason for doing so is that this parameter is not correlated with $\omega$ and could be precomputed from the contacts of observed events for given pole and semimajor axis (see Section 2.3.1). Moreover, in an eccentric orbit assumption, the value of $L_0$ derived for circular orbit can be used as the first approximation, without the need to know a location of orbit’s pericenter.

### 2.4.3 Brightness of binary asteroid

The total brightness of the system as seen by the observer is defined as

$$I = D(\Delta, R, \alpha) \left( \sum_{i=1}^{N_1} A_{1,i} o_{1,i} r_{1,i} \cos \epsilon_{1,i} + \sum_{i=1}^{N_2} A_{2,i} o_{2,i} r_{2,i} \cos \epsilon_{2,i} \right). \quad (37)$$

Function $D$ contains geo- and heliocentric distance and phase angle factors. It is omitted in an analysis since the observed data can be easily reduced to unit distances and to given phase angle. The expression in parentheses is a sum of individual contributions from $N_1$ and $N_2$ facets of the primary and secondary, respectively. $A_{1,i}$ and $A_{2,i}$ are areas of the facets, $r_{1,i}, r_{2,i}$ are bidirectional reflectances defined by scattering law (see Section 2.4.4), and $\epsilon_{1,i}, \epsilon_{2,i}$ are angles of emergence of the facets. Factors $o_{1,i}, o_{2,i}$ represent a visibility, which is zero if the facet is not visible or not illuminated due to its orientation, i.e., $\mathbf{n}_f \cdot \mathbf{s} \leq 0$ or $\mathbf{n}_f \cdot \mathbf{h} \leq 0$, where $\mathbf{n}_f$ is an external normal vector of the facet, and $\mathbf{s}$ and $\mathbf{h}$ are unit vectors in the directions to the Sun and to the Earth, respectively.

The occultations or eclipses by the other body are examined using simple ray-tracing code, restricted here for simplicity to the occultation, while the other case is mathematically equivalent. First, the silhouette of the occulting body in viewing direction is constructed in a plane perpendicular to the line of sight. Center of each
Figure 8: Schematic view demonstrating the determination of visible fraction of facet’s surface. See text for details.

The facet of the occulted body is then projected to this plane, and its shortest distance from the silhouette, \( d \), is computed (see Fig. 8). If the center is inside the silhouette, then the value of \( d \) is taken as negative. A precomputed "effective diameter" of the facet defined as \( l = \sqrt{4A/\pi} \), where \( A \) is the area of the facet, is projected to the perpendicular plane as \( l_p = l \cos \epsilon \). This value is then compared with the distance \( d \) to obtain the facet’s visibility \( o \):

- if \( d \geq l_p/2 \), then \( o = 1 \),
- if \( d \leq -l_p/2 \), then \( o = 0 \),
- if \( -l_p/2 < d < l_p/2 \), then \( o = (d + l_p/2)/l_p \).

2.4.4 Scattering laws

Three photometric laws are generally used for modeling asteroid lightcurves: Lommel-Seeliger law (see, e.g., Kaasalainen and Torppa, 2001), and two Hapke’s laws for macroscopically smooth and rough surface (Bowell et al., 1989). One of these laws can be selected in my model and they are described in this section for completeness.

The bidirectional reflectance \( r \) has a general form

\[
r = J(\mu, \mu_0, \alpha) \varpi,
\]

where \( J \) and \( \varpi \) are the scattering law and albedo, respectively, \( \mu \) and \( \mu_0 \) are cosines of angles of emergence, \( \epsilon \), and incidence, \( \iota \), respectively,

\[
\mu = \cos \epsilon = \mathbf{h} \cdot \mathbf{n_r},
\]
\[
\mu_0 = \cos \iota = \mathbf{s} \cdot \mathbf{n_r},
\]

and \( \alpha \) is a phase angle (i.e., an angle between directions of an incident and an emerging light).
The simplest scattering law is Lambertian,
\[ J_L = \mu_0, \]
which applies to surfaces that reflect light diffusively in all directions. It happens for high-albedo surfaces, where multiple scattering is significant. Since most asteroids are low-albedo objects, the Lambertian law is not suitable for them.

**Lommel-Seeliger law**

A good approximation of scattering of a low-albedo surface, where only single scattering is significant, is the Lommel-Seeliger law:
\[ J_{LS} = \frac{\mu_0}{\mu_0 + \mu}. \]

A characteristic property of the Lommel-Seeliger (L-S) law is a uniform surface luminosity at zero phase, regardless of the shape of the body. At zero phase, \( \mu_0 = \mu \) for any surface element, and so \( J_{LS} \) is constant.

A rigorous derivation of the L-S law can be found in Hapke (1993), thus only a brief explanation of formula (40) is presented below. The L-S law describes the reflectance of a semiinfinite particulate medium, which is homogeneous and isotropical in terms of absorption and scattering properties. The irradiance \( I_e \) entering the surface of the medium under the angle of incidence \( i \) is attenuated by the particles between the surface and some volume element \( dV \) by a factor \( e^{-Ez/\mu_0} \), where \( E \) is an extinction (i.e., absorption + scattering) coefficient, \( z \) is depth (i.e., vertical coordinate positive in the direction from upper empty space to the medium), and \( z/\mu_0 \) is the optical path (see Fig. 9). As was assumed above, the light is scattered only once in \( dV \). The scattered irradiance propagating under the angle \( \epsilon \) is similarly attenuated by a factor \( e^{-Ez/\mu} \). The total irradiance emerging from the surface \( I_e \) is an integral of the scattered contributions over all volume elements \( dV \) in the direction \( \epsilon \);
\[ I_e \sim I_i \int_G e^{-Ez/\mu_0} e^{-Ez/\mu} dV, \]
where \( G \) is a semiinfinite cylinder in the direction \( \epsilon \) with a base \( dA \). The volume element \( dV \) can be expressed as \( dV = (dA \, dh) = (dA \, dz/\mu) \), and the above integral could be rewritten as
\[ I_e \sim I_i \int_{z=0}^{\infty} e^{-Ez/\mu_0} e^{-Ez/\mu} \frac{dz}{\mu}. \]

The evaluation of this integral gives
\[ I_e \sim I_i \frac{\mu_0}{E \frac{\mu}{\mu + \mu_0}}, \]
which is, except for the omitted constants, the Lommel-Seeliger law.

\(^{13}\)In rigorous formulations of all laws presented in this subsection, the right sides of the equations contain also scaling and optical constants. Since in modeling of asteroidal lightcurves, only relative intensities/magnitudes are used, the factors are omitted for simplicity.
Hapke’s law for macroscopically smooth surface

More general Hapke’s law introduces three corrections to Lommel-Seeliger law: anisotropic scattering, multiple scattering, and effects of blocking and shadowing by individual particles resulting in the so-called opposition effect.

The anisotropic scattering is described by a single-particle scattering function \( p(\alpha) \) and it could be incorporated directly to the L-S law as

\[
J = \frac{\mu_0}{\mu_0 + \mu} p(\alpha).
\]  
(41)

For irregular particles, from which planetary and asteroidal regoliths are made, empirical scattering functions are frequently used (in that case, of course, \( p(\alpha) \) is a scattering function averaged over large number of individual particles). The simplest first-order expansion is \( p(\alpha) = 1 + b \cos \alpha \), where \( b \) is constant. Henyey and Greenstein (1941) introduced the empirical phase function as

\[
p(\alpha) = \frac{(1 - g^2)}{(1 + 2g \cos \alpha + g^2)^2},
\]  
(42)

where \( g \) is the asymmetry factor.

A contribution of the multiply scattered light to the irradiance emerging from the surface can be linearly incorporated to the scattering law as

\[
J = J_S + J_M,
\]  
(43)

where \( J_S \) and \( J_M \) are single (e.g., Lommel-Seeliger) and multiple scattering components, respectively. Hapke (1981) approximated \( J_M \) as

\[
J_M = \frac{\mu_0}{\mu_0 + \mu} [H(\mu_0)H(\mu) - 1],
\]  
(44)

where

\[
H(x) = \frac{1 + 2x}{1 + 2x(1 - x)^{\frac{1}{2}}},
\]
The third correction to the Lommel-Seeliger law is the opposition effect. In the previous derivation of the Lommel-Seeliger law, the factor $e^{-Kz/\mu_0}$ can be regarded as the probability that the incident irradiance will penetrate to the depth $z$. Similarly, $e^{-Kz/\mu}$ represent the probability that the light emerging from the element $dV$ will reach the surface of the medium. The derivation also assumes that the two probabilities are independent one of each other. However, they are not independent if the phase angle is small. If an incident ray is able to penetrate to a given depth without being blocked by any particle, then the ray scattered in this depth backward to the source will be able to reach the surface without being blocked too. This preferential escape of light causes a surge in brightness of particulate medium for phase angles below $\sim 10$ degrees. Although an analytic correction of the single scattering law can be derived (Hapke, 1993), the same author present a simple approximation in the form of

$$J = \frac{\mu_0}{\mu_0 + \mu} \left\{1 + B(\alpha)\right\}, \quad (45)$$

where

$$B(\alpha) = \frac{B_0}{1 + \frac{1}{h} \tan \frac{\alpha}{2}}$$

and $B_0$ and $h$ are an amplitude and an angular width of the opposition effect.

The directional effects of phase function $p(\alpha)$ are averaged out for multiply scattered light, whose angular distribution is therefore close to isotropic. For that reason, the term $J_M$ in the scattering law is relatively insensitive to $p(\alpha)$. Similarly, the preferential escape applies only to the light scattered only once, and the opposition effect (45) does not influence $J_M$. Combining the equations (41), (44), and (45), the scattering law can be therefore written as

$$J_{HS} = \frac{\mu_0}{\mu_0 + \mu} \left\{1 + B(\alpha)\right\} p(\alpha) + H(\mu_0)H(\mu) - 1, \quad (46)$$

which is Hapke’s law for macroscopically smooth surface.

**Hapke’s law for macroscopically rough surface**

As shown by Hapke (1984), any bidirectional reflectance function of a smooth surface could be corrected for macroscopic roughness, using some assumptions and approximations. The correction involves the so-called shadowing function $S(\iota, \varepsilon, \varphi)$ and additional arbitrary parameter, the mean slope angle of surface facets $\bar{\theta}$. $S$ is dependent on azimuth angle $\varphi$, which is an angle between incidence and emergence vectors projected onto the facet’s surface, $\cos \varphi = (\cos \alpha - \cos \iota \cos \varepsilon) / (\sin \iota \sin \varepsilon)$. The macroscopically rough surface is assumed to be made of facets with a Gaussian distribution in $\tan \bar{\theta}$, where $\bar{\theta}$ is the angle between the normal to the facet and the normal to the mean surface.

The macroscopic roughness has two major effects to the photometric law: 1) shadows are cast on the surface by raised parts or some parts of the surface are hidden to the observer. A decrease of the total flux caused by shadowed or hidden parts is described by the shadowing function $S(\iota, \varepsilon, \varphi)$. 2) At increasing angles of incidence or emergence, facets that are tilted away from the source or the observer
tend to be in shadow or hidden. Therefore, the facets which are illuminated and visible tend to be tilted preferentially toward the source or the observer. The average (effective) angles of incidence or emergence of the surface consisting of such facets are then smaller – this is accounted for by replacing \( \mu \) and \( \mu_0 \) by effective cosines \( \mu' \) and \( \mu'_0 \), respectively, in the photometric law. The resulting form of the Hapke’s law for macroscopically rough surface is

\[
J_{HR} = \frac{\mu'_0}{\mu'_0 + \mu'} \left[ \{1 + B(\alpha)\} p(\alpha) + H(\mu'_0)H(\mu') - 1 \right] S(\iota, \epsilon, \varphi).
\] (47)

\( S, \mu' \) and \( \mu'_0 \) have slightly different forms depending on the relative values of angles of incidence \( \iota \) and emergence \( \epsilon \).

For \( \iota \leq \epsilon \):

\[
S(\iota, \epsilon, \varphi) = \frac{\mu'(\iota, \epsilon, \varphi)}{\mu'(0, \epsilon, 0)} \frac{\mu_0}{\mu'_0(\iota, 0, \pi)} C(\theta) \left\{ 1 - f(\varphi) \left[ 1 - C(\theta) \frac{\mu_0}{\mu'_0(\iota, 0, \pi)} \right] \right\}^{-1},
\]

where

\[
\mu'_0(\iota, \epsilon, \varphi) = C(\theta) \left[ \mu_0 + \sin \iota \tan \theta \frac{\cos \varphi E_2(\epsilon) + \sin^2(\varphi/2)E_2(\iota)}{2 - E_1(\epsilon) - (\varphi/\pi)E_1(\iota)} \right]
\]

and

\[
\mu'(\iota, \epsilon, \varphi) = C(\theta) \left[ \mu + \sin \epsilon \tan \theta \frac{E_2(\epsilon) - \sin^2(\varphi/2)E_2(\iota)}{2 - E_1(\epsilon) - (\varphi/\pi)E_1(\iota)} \right].
\]

For \( \iota > \epsilon \):

\[
S(\iota, \epsilon, \varphi) = \frac{\mu'(\iota, \epsilon, \varphi)}{\mu'(0, \epsilon, \pi)} \frac{\mu_0}{\mu'_0(\iota, 0, 0)} C(\theta) \left\{ 1 - f(\varphi) \left[ 1 - C(\theta) \frac{\mu}{\mu'(0, \epsilon, \pi)} \right] \right\}^{-1},
\]

where

\[
\mu'_0(\iota, \epsilon, \varphi) = C(\theta) \left[ \mu_0 + \sin \iota \tan \theta \frac{E_2(\iota) - \sin^2(\varphi/2)E_2(\epsilon)}{2 - E_1(\iota) - (\varphi/\pi)E_1(\epsilon)} \right]
\]

and

\[
\mu'(\iota, \epsilon, \varphi) = C(\theta) \left[ \mu + \sin \epsilon \tan \theta \frac{\cos \varphi E_2(\iota) + \sin^2(\varphi/2)E_2(\epsilon)}{2 - E_1(\iota) - (\varphi/\pi)E_1(\epsilon)} \right].
\]

The other functions used in the above formulas are defined as

\[
f(\varphi) = e^{-2 \tan(\varphi/2)},
\]

\[
C(\theta) = (1 + \pi \tan \theta)^{-1/2},
\]

\[
E_1(x) = e^{-(2/\pi) \cot \theta \cot x},
\]

\[
E_1(x) = e^{-(1/\pi) \cot^2 \theta \cot^2 x}.
\]

The formulas used in Hapke’s laws contain three or four constants, which are usually fitted as free parameters: \( B_0 \) – amplitude of the opposition effect, \( h \) – angular width of the opposition effect, \( g \) – asymmetry factor in the phase function, and \( \tilde{\theta} \) – mean slope angle of surface facets. The timesaving Lommel-Seeliger law is used for model lightcurves of all binaries studied in this work. In several cases, Hapke’s law for macroscopically rough surface was also tested and it was found that parameters of obtained best-fit solution are only weakly sensitive to the photometric law used (see Section 4.2).
2.4.5 Interpolation of synthetic lightcurve

Since the overall shape of synthetic lightcurve is quite smooth at almost all points, it would be useless and time-consuming to compute the total brightness of binary at epochs of all observed data points, especially if the data are dense. Instead, it is more effective to divide timespans of observations into an equidistant grid, to evaluate the brightness only at this grid, and to interpolate the values at the points within. The dividing interval was chosen to be 0.01 days for all modeled systems (see Section 4.4 for discussion) and an example of the interpolated synthetic lightcurve is shown at Fig. 10.

However, at the time instants of events’ contacts the condition of lightcurve smoothness is not fulfilled, and thus their epochs have to be included in the grid. The epochs are computed from true anomalies of radius vectors of secondary, solved for spherical approximation of the bodies’ shapes using algorithm described in Section 2.2. On the other hand, the intervals where brightness of the system is constant, i.e., intervals outside the mutual events and the flat bottoms of the secondary events for spherical bodies, don’t need to be divided into smaller grid at all.

It should be emphasized that, because the interpolation is applied to the specific synthetic lightcurve and not to the real data, all parameters needed for this evaluation (i.e., location of pericenter, eccentricity, orbital pole, etc.) are given in this case.

2.5 Deriving binary parameters – the inverse problem

The inverse problem entails the derivation of binary system parameters from the observed lightcurve data. More specifically, it is equivalent to finding all plausible minima of a function (root-mean-square)

$$\text{RMS}(\mathbf{B}, \Delta m_1, \ldots, \Delta m_P) = \sqrt{\frac{\sum_{i=1}^{N} (o_i - c_i)^2}{N}},$$  \hspace{1cm} (48)

where \( \mathbf{B} \) is vector of fitted parameters, \( \Delta m_1, \ldots, \Delta m_P \) are offsets (in magnitudes) of lightcurve sequences that are not absolutely calibrated (if all the sequences are calibrated, then \( P = 1 \)), while \( o_i \) and \( c_i \) are observed and calculated magnitudes, respectively, of \( N \) data points in the lightcurve. The term lightcurve sequence refers in this work to a subset of all photometric measurements with common set of comparative stars within the lightcurve.

Usually, many authors use the \( \chi^2 \) statistics to minimization and errors estimation. This approach assumes that the observational errors are random and that the model gives a good description of the data, but neither is guaranteed here. The observed data itself contain systematic errors due to the interfering stars and changes in observational conditions and, more importantly, the model is only approximation and thus produces systematic deviations between synthetic and observed data. Therefore, an arbitrary function of \( (\sum_{i=1}^{N} (o_i - c_i)^2) \) or \( (\sum_{i=1}^{N} |o_i - c_i|) \) can be used for minimization. I choose the RMS function for its explicit meaning.

Since all primaries of studied binaries are approximated by rotationally symmetric shape, only the long-period component of their lightcurves are modeled in this work. These components are hereafter referred to as lightcurves for simplicity.
Figure 10: Example of interpolated synthetic lightcurve of two orbiting spheres. The interpolation interval is set to be equal or smaller than 0.01 d. The points where magnitude was computed are indicated by ticks at the bottom of the diagram. The contacts (time instants at which partial or total events begin or end) are indicated by vertical lines, the meanings of abbreviations are as follows: \textit{pepb} – beginning of partial phase of secondary’s shadow transit across the primary, \textit{popb} – beginning of partial phase of secondary’s transit in front of the primary, \textit{pefb} – beginning of full phase of secondary’s shadow transit across the primary, \textit{pofb} – beginning of full phase of secondary’s transit in front of the primary, \textit{pefe} – end of full phase of secondary’s shadow transit across the primary, \textit{pofe} – end of full phase of secondary’s transit in front of the primary, \ldots, \textit{sepb} – beginning of secondary’s partial eclipse, \textit{sob} – beginning of secondary’s partial occultation, \textit{setb} – beginning of secondary’s total eclipse, etc.
In the simplest model with spherical bodies orbiting each other on the circular orbit, the list of fitted parameters B consists of five parameters: ratio of component diameters $D_s/D_p$, the mutual orbit semimajor axis-to-primary diameter ratio $a/D_p$ and its pole in ecliptic coordinates $\lambda_p, \beta_p$, the sidereal period $P_{\text{sid}}^{\text{orb}}$, and the secondary’s argument of mean length $L_0$ at a given epoch $T_0$.

With increasing complexity the number of fitted parameters also increases. They could include the eccentricity and the argument of pericenter of the mutual orbit, parameters describing the shape, rotational periods and spin poles of both components, etc.

The minimization process is divided into several steps.

I. First, an initial value (or its lower limit) of $D_s/D_p$ is derived from the lightcurve using Eq. 30, the contacts of events are identified visually in the lightcurve (see Section 2.2), and the synodic orbital period $P_{\text{syn}}^{\text{orb}}$ is derived from the periodicity of the events.

II. An equidistant grid in $\beta_p$ and $\lambda' = \lambda_p \cos \beta_p$\textsuperscript{14} is constructed in the next step. For each point of the grid, a reasonable interval of the orbit semimajor axis $a/D_p$\textsuperscript{15} is sampled and for each combination of $\beta_p, \lambda_p$ and $a/D_p$ the sidereal period $P_{\text{sid}}^{\text{orb}}$ with the argument of mean length $L_0$ of the secondary are computed using methods described in Sections 2.2 and 2.3.1. The value of $a/D_p$ with the best fit of Eq. 33 to the epochs of contacts identified in the lightcurve is then selected as the most probable value. If some of the observed contact couldn’t occur for given pair of $\lambda_p, \beta_p$ and any of the tested values of $a/D_p$, the pair is excluded from further analysis.

III. The above-presented process serves to create a map of initial values of parameters $D_s/D_p, \lambda_p, \beta_p, a/D_p, P_{\text{sid}}^{\text{orb}}$ and $L_0$ (see Fig. 6). Starting from each point of this map, a local minimum of the RMS function is found using the Nelder and Mead Simplex Algorithm (see Section 2.5.3) for spherical components and circular orbit approximation.

IV. In order to make subsequent computation faster, only a subset of reached local minima is selected for further analysis. The minima are selected from those with the lowest RMS to those with the largest, while a distance in $\lambda_p, \beta_p$ space of each selected minimum from all previous has to be at least 10 degrees.

V. For each set of parameters of these selected minima a grid in $e$ and $\omega$ is created. This grid serves as initial points for the next minimization using the simplex algorithm, where fitted parameters are $B = D_s/D_p, \lambda_p, \beta_p, a/D_p, P_{\text{sid}}^{\text{orb}}, L_0, e, \omega$.

VI. The same selection process as in the step IV is then applied to all found local minima.

VII. A grid in $A_p/C_p$ and $A_s/C_s$ is then created for all selected minima and a minimization is restarted from this grid in order to derive the flattening of the primary and the elongation of the secondary. Fitted parameters are $B = D_s/D_p, \lambda_p, \beta_p, a/A_p, P_{\text{sid}}^{\text{orb}}, L_0, e, \omega, A_p/C_p, A_s/C_s, A_s/A_p$. An initial values of the component size ratio and semimajor axis are set as $A_s/A_p = D_s/D_p$ and $a/A_p = 2(a/D_p)$, respectively.

\textsuperscript{14}This parametrization ensures that an equal area is belonging to each point of the $\lambda_p, \beta_p$ grid.

\textsuperscript{15}The lower boundary of the interval is $(a/D_p)_{\text{min}} = 1/2 + (D_s/D_p)/2$, while the upper value can be guessed from Kepler’s third law using synodic orbital period and reasonable upper limit on bulk density (see Eq. 49).
VIII. Finally, synthetic lightcurves are constructed for all local minima found in the step VII. Since differences between observed data and the synthetic lightcurves are dominated by model idealizations and/or systematic errors in the data, all the synthetic lightcurves are compared with the data visually and the most plausible solutions are selected.

During the minimization, the following constraints are put on the parameters to avoid unrealistic or implausible states:

- The largest diameter of the secondary is always forced to be smaller than or equal to the largest diameter of the primary.
- If the primary is assumed to be an oblate spheroid, $A_p/C_p$ and $B_p/C_p$ are forced to be $\geq 1$.
- If the secondary is assumed to be a prolate spheroid, $A_s/C_s$ is forced to be $\geq 1$.
- The combination of mutual orbit semimajor axis and dimensions of primary and secondary are forced to not allow the two components to touch or intersect at pericenter.

2.5.1 Bulk density

Assuming that the bulk density is the same for both components, Kepler’s third law for spherical bodies can be rewritten in the form

$$\frac{24\pi}{GP_{\text{orbit}}^2} \left[ \left( \frac{D_s}{D_p} \right)^3 + 1 \right]^{-1} \left( \frac{a}{D_p} \right)^3 = \rho, \quad (49)$$

where $G$ is the gravitational constant and $\rho$ is the bulk density. Since real shapes of asteroids definitely are not spherical, using the above formula without caution is the largest source of error in derived densities and in many cases lead to unrealistically low values. For real bodies with arbitrary shapes, the term $(D_s/D_p)^3$ should be replaced with a ratio of volumes of secondary and primary, $V_s/V_p$.

If the primary shape is assumed to be homogenous oblate ellipsoid and considering the effect of the oblateness up to the $J_2$ term (the first zonal harmonic coefficient in the gravitational potential expansion), Kepler’s third law can be modified as follows (see Rossi et al., 1999 or Murray and Dermott, 1999, page 268):

$$\frac{4\pi a^3}{GP_{\text{orbit}}^2 (V_p + V_s)} \epsilon = \rho, \quad (50)$$

where

$$\epsilon = 1 - \frac{3}{10} \frac{A_p^2 - C_p^2}{a^2} \quad (51)$$

The spherical or ellipsoidal approximation of asteroidal shapes always overestimates their real volumes. Thus, the bulk densities derived using formulas (49) or (50) should be taken as their lower limits.
2.5.2 Assessment of parameters’ errors

A special emphasis is given to assess the errors of parameters of plausible solutions. Due to the model idealizations and systematic errors, formal $\chi^2$-error estimates gave unrealistically small uncertainties. Thus, all fitted parameters were stepped in a reasonable interval around their best-fit solutions values and held fixed, while the others were fitted by the simplex algorithm. Synthetic lightcurves were constructed for each result of this fitting and visually compared with the data. Parameters respective to the plausible selected curves represent 3-σ range of the solutions.

In addition, for easier comparison of orbital poles obtained for different apparitions and with radar-derived poles, $\lambda_p$ and $\beta_p$ were stepped in a two-dimensional grid and ranges of possible values are shown as plots in Section 3.

The error of the bulk density can’t be estimated using the above method, because the density is not among directly fitted parameters but it is derived from them. The range of possible densities can be obtained by varying these parameters separately, but in order not to miss their combination leading to extremal values of density, the following method was developed:

An auxiliary parameter $\rho_0$ is stepped in a reasonable interval, usually [0.5, 4.0]. For every value of $\rho_0$, the parameters describing the binary system are varied in order to reach a minimum of a function

$$X = \text{RMS} \cdot (1 + w|\rho(B) - \rho_0|),$$

(52)

where RMS is defined by Eq. (48) and $\rho$ is considered as a function of varied parameters defined by Eq. (50). Several values of the weighting factor $w$ in a range from 0.5 to 2.0 were used. The purpose of this procedure is to obtain a parameter set not just with the value of $\rho$ close to or equal to stepped parameter $\rho_0$, but also with the best possible agreement to the observed data. After each fitting, synthetic curve was constructed using the resulting parameters, compared visually with the observed data, and the 3-σ range of $\rho$ was extended in the case of a good match.

2.5.3 Nelder and Mead Simplex Algorithm

The Nelder and Mead Simplex Algorithm (Buchanan and Turner, 1992) is one of the so-called simplex optimization algorithms, whose advantage is that only values of a fitted function but not its derivatives have to be evaluated. In this section, the algorithm is described in its general form as a minimization of n-dimensional function $f(x_1, x_2, \ldots, x_n)$.

The simplex in $\mathbb{R}^n$ is defined as a set of $n + 1$ points $x_0, x_1, \ldots, x_n$ forming a non-degenerated polygon (triangle for $\mathbb{R}^2$, tetrahedron for $\mathbb{R}^3$, etc.). Starting from initial simplex, the function $f$ is evaluated at all simplex vertices. The values are then compared and, based on simple rules, one of the vertices is replaced with the new point. Thus, the simplex is changing shape and position in order to move closer to the minimum. These rules are described below and schematically shown in Fig. 11.

1. Denote $G, H$ and $S$ vertices with the largest, second largest and the smallest functional values $f_G, f_H$ and $f_S$, respectively. Compute a center of simplex’s
face which don’t include $G$:
\[
X = \frac{1}{n} \left( \sum_{i=0}^{n} x_i - G \right).
\]  
(53)

2. Each iteration starts with a reflection step, where the vertex $G$ is reflected through $X$ to new point $R$:
\[
R = X + a(X - G).
\]  
(54)

Denote functional value in $R$ as $f_R$. The range of a reflection coefficient $a$ could be $0 < a \leq 1$, but the most common value is 1.

3. If $f_R < f_S$, then an expansion step in the same direction creates new point
\[
E = X + b(R - X),
\]  
(55)

where an expansion coefficient $b > 1$, usually $b = 2$.

4. If $f_E < f_R$, then vertex $G$ in the original simplex is replaced with $E$ and the next iteration starts from the step 1.

5. If $R$ provides an improvement with respect to the other vertices but not to $S$ ($f_H > f_R \geq f_S$) at the step 2., then $G$ is replaced with $R$.

6. In a case when $f_G > f_R \geq f_H$, the point $R$ provides an improvement with respect to the point $G$, but becomes the worst point in new simplex. Therefore, the next iteration would evoke the reflection heading back to $G$, and even directly into $G$ for $a = 1$. In that case a contraction step creates a new point outward from the current simplex:
\[
C = X + c(R - X).
\]  
(56)

If the reflected point $R$ doesn’t provide an improvement at all ($f_R \geq f_G$), the contraction is inward to
\[
C = X + c(G - X).
\]  
(57)

The range of contraction coefficient is $0 < c < 1$, usually $c = 1/2$. If $f_C < f_G, f_R$, then $G$ is replaced with $C$ and the next iteration starts from the step 1.

7. If any of the above steps does not provide an improvement, then the whole simplex is rescaled to half size towards $S$:
\[
x_i = \frac{x_i + S}{2} \quad (i = 0 \ldots n).
\]  
(58)

8. The optimization process is terminated when for all vertices of the simplex $x_0 \ldots x_n$ the following condition is accomplished:
\[
\forall k = 1 \ldots n : \max_{i \neq j} |x_{ik} - x_{jk}| < \varepsilon_k,
\]  
(59)

where $\varepsilon_k$ is chosen tolerance for k-th parameter. The coordinates of the searched minimum are then corresponding components of $S$. 

37
input: $x_0 \ldots x_n$

$G$ – vertex with the largest func. value.
$H$ – vertex with second largest func. value
$S$ – vertex with the smallest func. value

$X = \frac{1}{n} \left( \sum x_i - G \right)$

$R = X + a(X - G)$

$\begin{align*}
    f_R < f_G & \quad \text{yes} \quad G = R \\
    f_R < f_S & \quad \text{yes} \quad E = X + b(R - X) \\
    f_E < f_R & \quad \text{yes} \quad G = E \\
    f_R \geq f_H & \quad \text{yes} \quad C = X + c(G - X) \\
    f_C < f_H & \quad \text{yes} \quad G = C \\
    \forall k = 1 \ldots n: \\
    & \quad \max_{i \neq j} |x_{ik} - x_{jk}| < \varepsilon_k \\
    & \quad \text{no} \quad x_i = (x_i + S)/2
\end{align*}$

output: $S$

Figure 11: Schematic illustration of the Nelder and Mead simplex algorithm. See text for explanation of symbols.
<table>
<thead>
<tr>
<th>Object</th>
<th>Apparition</th>
<th>Time span (d)</th>
<th>Geo. arc (deg)</th>
<th>Hel. arc (deg)</th>
<th>Phase angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(88710)</td>
<td>2001 SL₉</td>
<td>8</td>
<td>12</td>
<td>5</td>
<td>1 – 8</td>
</tr>
<tr>
<td>(35107)</td>
<td>1991 VH</td>
<td>32</td>
<td>19</td>
<td>20</td>
<td>18 – 38</td>
</tr>
<tr>
<td></td>
<td>2003 Feb</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>24 – 30</td>
</tr>
<tr>
<td>(65803)</td>
<td>Didymos</td>
<td>29</td>
<td>38</td>
<td>29</td>
<td>2 – 19</td>
</tr>
<tr>
<td>(3671)</td>
<td>Dionysus</td>
<td>31</td>
<td>21</td>
<td>28</td>
<td>39 – 68</td>
</tr>
<tr>
<td>(66391)</td>
<td>1999 KW₄</td>
<td>2000 May–Jun*</td>
<td>35</td>
<td>90</td>
<td>62 – 76</td>
</tr>
<tr>
<td>(66063)</td>
<td>2000 DP₁₀⁷</td>
<td>8</td>
<td>19</td>
<td>7</td>
<td>30 – 38</td>
</tr>
<tr>
<td></td>
<td>2002 Sep*</td>
<td>3</td>
<td>6</td>
<td>1</td>
<td>3 – 8</td>
</tr>
<tr>
<td></td>
<td>2003 Sep</td>
<td>9</td>
<td>33</td>
<td>5</td>
<td>12 – 33</td>
</tr>
<tr>
<td></td>
<td>2004 Sep</td>
<td>11</td>
<td>28</td>
<td>6</td>
<td>30 – 36</td>
</tr>
</tbody>
</table>

Notes. The fourth and the fifth columns give the sky arc spanned with respect to the Earth and to the Sun, respectively. Time span, phase angle and arcs cover only lightcurve sequences at which at least part of a mutual event was observed.

* Denotes apparitions with data of low quality or too short time span, that were only checked for consistency with solutions derived from another apparition(s).

3 Results

The total number of eight near-Earth binary asteroids were modeled, some of them in multiple apparitions. The binaries are listed in Table 3 with their observational circumstances, while the best-fit values of modeled parameters and their errors are summarized in Table 4. Table 5 lists types of events occurring in their lightcurves, determined from estimations of their mutual orbits’ normals.

Although solutions derived from radar data or from optical data obtained during other apparitions exist for some cases, the presented results are based solely on photometric data taken during given apparitions. This helps to better understand the effects influencing the accuracy of derived parameters, mainly of the position of the mutual orbit’s pole, which will be crucial when larger sample of binaries data will be available and a debiasing of their orbital pole distribution will be possible. Indeed, the results of other works are cited as well.

This section is divided into two subsections. In the first one the modeled systems are described in detail, while in the second the dependence of uncertainties of some of the modeled parameters on the observational circumstances is studied.

All dates and times used in this section are in the UTC and have been corrected for light-travel time.
Table 4: Parameters of binary NEAs estimated with the numerical models, with $3 - \sigma$ errors.

<table>
<thead>
<tr>
<th>Object (apparition) Solution</th>
<th>$\omega$ (°)</th>
<th>$I_0$ (°)</th>
<th>epoch (MJD)</th>
<th>$A_2/A_0$</th>
<th>$A_2/C_0$</th>
<th>$A_2/C_2$</th>
<th>$\rho$ (g cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996 FG$_3$ (1998) I.</td>
<td>79.4$^{+0.5}_{-0.4}$</td>
<td>43.8$^{+0.6}_{-0.6}$</td>
<td>51150.6792</td>
<td>0.33$^{+0.07}_{-0.06}$</td>
<td>1.4$^{+0.9}_{-0.9}$</td>
<td>1.4$^{+0.5}_{-0.5}$</td>
<td></td>
</tr>
<tr>
<td>(88710) 2001 SL$_9$ (2001) I.</td>
<td>330.4$^{+0.1}_{-0.1}$</td>
<td>138.7$^{+0.1}_{-0.1}$</td>
<td>52192.4316</td>
<td>0.25$^{+0.02}_{-0.02}$</td>
<td>1.0$^{+0.2}_{-0.2}$</td>
<td>1.0$^{+0.2}_{-0.2}$</td>
<td></td>
</tr>
<tr>
<td>(35107) 1991 VH (1997) I.</td>
<td>146.2$^{+0.2}_{-0.2}$</td>
<td>93.7$^{+0.2}_{-0.2}$</td>
<td>50050.9547</td>
<td>0.31$^{+0.1}_{-0.1}$</td>
<td>1.7$^{+0.9}_{-0.9}$</td>
<td>1.7$^{+0.9}_{-0.9}$</td>
<td></td>
</tr>
<tr>
<td>II.</td>
<td>302.4$^{+0.1}_{-0.1}$</td>
<td>346.0$^{+0.1}_{-0.1}$</td>
<td>&quot;</td>
<td>0.42$^{+0.03}_{-0.03}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>III.</td>
<td>60.1$^{+0.1}_{-0.1}$</td>
<td>101.4$^{+0.1}_{-0.1}$</td>
<td>&quot;</td>
<td>0.37$^{+0.04}_{-0.04}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>IV.</td>
<td>352.7$^{+0.6}_{-0.6}$</td>
<td>35.0$^{+0.6}_{-0.6}$</td>
<td>&quot;</td>
<td>0.42$^{+0.02}_{-0.02}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>V.</td>
<td>15.4$^{+0.0}_{-0.0}$</td>
<td>315.1$^{+0.0}_{-0.0}$</td>
<td>&quot;</td>
<td>0.37$^{+0.08}_{-0.08}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>(35107) 1991 VH (2003) I.</td>
<td>283.3$^{+0.3}_{-0.3}$</td>
<td>233.4$^{+0.3}_{-0.3}$</td>
<td>52671.8388</td>
<td>0.26$^{+0.02}_{-0.02}$</td>
<td>2.0$^{+0.2}_{-0.2}$</td>
<td>1.0$^{+0.2}_{-0.2}$</td>
<td></td>
</tr>
<tr>
<td>II.</td>
<td>352.7$^{+0.6}_{-0.6}$</td>
<td>226.2$^{+0.6}_{-0.6}$</td>
<td>&quot;</td>
<td>0.25$^{+0.03}_{-0.03}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>III.</td>
<td>206.1$^{+0.1}_{-0.1}$</td>
<td>65.7$^{+0.1}_{-0.1}$</td>
<td>&quot;</td>
<td>0.26$^{+0.03}_{-0.03}$</td>
<td>2.1$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>IV.</td>
<td>189.6$^{+0.1}_{-0.1}$</td>
<td>54.1$^{+0.1}_{-0.1}$</td>
<td>&quot;</td>
<td>0.38$^{+0.04}_{-0.04}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>(65803) Didymos (2003) I.</td>
<td>340.2$^{+0.1}_{-0.1}$</td>
<td>120.1$^{+0.1}_{-0.1}$</td>
<td>52963.8922</td>
<td>0.21$^{+0.01}_{-0.01}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>(3671) Dionysus (1997) I.</td>
<td>327.7$^{+0.1}_{-0.1}$</td>
<td>550.4$^{+0.1}_{-0.1}$</td>
<td>50545.9601</td>
<td>0.20$^{+0.01}_{-0.01}$</td>
<td>1.5$^{+0.1}_{-0.1}$</td>
<td>1.2$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>II.</td>
<td>305.7$^{+0.1}_{-0.1}$</td>
<td>350.7$^{+0.1}_{-0.1}$</td>
<td>&quot;</td>
<td>0.22$^{+0.01}_{-0.01}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td>1.0$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>III.</td>
<td>205.7$^{+0.1}_{-0.1}$</td>
<td>272.1$^{+0.1}_{-0.1}$</td>
<td>&quot;</td>
<td>0.22$^{+0.01}_{-0.01}$</td>
<td>1.2$^{+0.1}_{-0.1}$</td>
<td>1.2$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>(65891) 1999 K4 (2001) I.</td>
<td>264.1$^{+0.1}_{-0.1}$</td>
<td>129.4$^{+0.1}_{-0.1}$</td>
<td>52054.0386</td>
<td>0.51$^{+0.02}_{-0.02}$</td>
<td>1.0$^{+0.2}_{-0.2}$</td>
<td>1.0$^{+0.2}_{-0.2}$</td>
<td></td>
</tr>
<tr>
<td>2000 DP$_{107}$ (2000) I.</td>
<td>328.1$^{+0.1}_{-0.1}$</td>
<td>270.7$^{+0.1}_{-0.1}$</td>
<td>51812.1522</td>
<td>0.44$^{+0.01}_{-0.01}$</td>
<td>1.2$^{+0.1}_{-0.1}$</td>
<td>1.2$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>II.</td>
<td>161.2$^{+0.1}_{-0.1}$</td>
<td>89.2$^{+0.1}_{-0.1}$</td>
<td>&quot;</td>
<td>0.37$^{+0.01}_{-0.01}$</td>
<td>1.6$^{+0.1}_{-0.1}$</td>
<td>1.1$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>(66063) 1998 RO$_4$ (2003) I.</td>
<td>195.9$^{+0.1}_{-0.1}$</td>
<td>264.6$^{+0.1}_{-0.1}$</td>
<td>52898.8447</td>
<td>0.45$^{+0.01}_{-0.01}$</td>
<td>1.8$^{+0.1}_{-0.1}$</td>
<td>1.8$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>II.</td>
<td>69.4$^{+0.1}_{-0.1}$</td>
<td>57.1$^{+0.1}_{-0.1}$</td>
<td>&quot;</td>
<td>0.39$^{+0.01}_{-0.01}$</td>
<td>2.2$^{+0.1}_{-0.1}$</td>
<td>1.7$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
<tr>
<td>(66063) 1998 RO$_4$ (2004) I.</td>
<td>154.7$^{+0.1}_{-0.1}$</td>
<td>215.6$^{+0.1}_{-0.1}$</td>
<td>53258.1605</td>
<td>0.59$^{+0.01}_{-0.01}$</td>
<td>1.8$^{+0.1}_{-0.1}$</td>
<td>1.8$^{+0.1}_{-0.1}$</td>
<td></td>
</tr>
</tbody>
</table>

Notes. The parameters are described in Sections 2.3.1, 2.4.1 and 2.4.2. RMS is a root mean square of residuals between observed and synthetic data, in magnitudes. $L_0$ is expressed for the given epoch.
<table>
<thead>
<tr>
<th>Object (apparition)</th>
<th>Solution</th>
<th>Events type</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996 FG₃ (1998)</td>
<td>I.</td>
<td>Both</td>
</tr>
<tr>
<td>(88710) 2001 SL₉ (2001)</td>
<td>I.</td>
<td>Both</td>
</tr>
<tr>
<td></td>
<td>II.</td>
<td>Both</td>
</tr>
<tr>
<td>(35107) 1991 VH (1997)</td>
<td>I.</td>
<td>Eclipses</td>
</tr>
<tr>
<td></td>
<td>II.</td>
<td>Occultations</td>
</tr>
<tr>
<td></td>
<td>III.</td>
<td>Occultations</td>
</tr>
<tr>
<td></td>
<td>IV.</td>
<td>Eclipses</td>
</tr>
<tr>
<td></td>
<td>V.</td>
<td>Eclipses</td>
</tr>
<tr>
<td></td>
<td>II.</td>
<td>Occultations</td>
</tr>
<tr>
<td></td>
<td>III.</td>
<td>Occultations</td>
</tr>
<tr>
<td></td>
<td>IV.</td>
<td>Eclipses</td>
</tr>
<tr>
<td>(65803) Didymos (2003)</td>
<td>I.</td>
<td>Both</td>
</tr>
<tr>
<td></td>
<td>II.</td>
<td>Both</td>
</tr>
<tr>
<td>(3671) Dionysus (1997)</td>
<td>I.</td>
<td>Eclipses (except the event at JD=2450580.50)</td>
</tr>
<tr>
<td></td>
<td>II.</td>
<td>Eclipses (except the event at JD=2452056.45)</td>
</tr>
<tr>
<td>(66391) 1999 KW₄ (2001)</td>
<td>I.</td>
<td>Eclipses</td>
</tr>
<tr>
<td></td>
<td>II.</td>
<td>Eclipses</td>
</tr>
<tr>
<td>(66063) 1998 RO₁ (2003)</td>
<td>I.</td>
<td>Both</td>
</tr>
<tr>
<td></td>
<td>II.</td>
<td>Both</td>
</tr>
</tbody>
</table>
3.1 Modeled binary systems

3.1.1 1996 FG₃

The binary nature of this near-Earth asteroid was discovered by Pravec et al. (2000) and by Mottola and Lahulla (2000). The results of both works were in agreement and they are summarized here. Pravec et al. (2000) derived $D_s/D_p = 0.31 \pm 0.02$, $a/D_p = 1.7 \pm 0.3$, $P_{\text{orb}} = (16.135 \pm 0.005)$ h and $e = 0.05 \pm 0.05$ from their lightcurve. Assuming that $D_s/D_p = 0.3$, $a/D_p = 1.55$, $P_{\text{orb}} = 16.135$ h, $\lambda_p = 196^\circ$ and $\beta_p = -80^\circ$ (they didn’t carry out the complete inverse search of parameters) they constructed a simple model with spherical bodies and a circular mutual orbit with synthetic lightcurve with a good match to observational data.

Mottola and Lahulla (2000) developed a numerical model consisting of two triaxial ellipsoids on a circular orbit. They determined $P_{\text{orb}}$ to be $16.15 \pm 0.02$ and obtained two solutions for orbital pole: $(\lambda_p = 282^\circ \pm 10^\circ$, $\beta_p = -87^\circ \pm 3^\circ)$ and $(\lambda_p = 262^\circ \pm 10^\circ$, $\beta_p = +75^\circ \pm 3^\circ)$, with the retrograde solution being more likely. The component shapes properties and size of the orbit were the same for both solutions. While Mottola and Lahulla used a slightly different parametrization from that used in this thesis, I converted their values to the system defined in Sections 2.4.1 and 2.4.2 for easier comparison, approximating the equatorial radius of primary as $A_p = \sqrt{AB}$, where $A, B$ are equatorial semiaxes of primary defined by Mottola and Lahulla. These converted parameters are as follows, and are in a good agreement with my results (see Table 4):

\begin{align*}
A_p/C_p & = 1.4 \pm 0.2, \\
A_s/C_s & = 1.4 \pm 0.2, \\
A_s/A_p & = 0.32 \pm 0.03, \\
a/A_p & = 2.9 \pm 0.1.
\end{align*}

Scheirich (2003) found two solutions from the Ondřejov data using spherical bodies on eccentric orbit: $(\lambda_p = 260^\circ \pm 50^\circ$, $\beta_p = -87^\circ \pm 5^\circ)$, and $(\lambda_p = 150^\circ \pm 4^\circ$, $\beta_p = +60^\circ \pm 8^\circ)$.

I reanalysed the data by Pravec et al. (2000) taken during 1998-12-03.7 to 1999-01-09.1 using the model of two-axis ellipsoids on the eccentric orbit described in Section 2.4 and found a single solution fitting the long-period component of the lightcurve satisfactorily. The parameters of this solution are summarized in Table 4, with their errors obtained using procedure described in Section 2.5.2. The Lommel-Seeliger photometric law was used in the modeling and all lightcurve sequences of the data were taken as absolutely calibrated (i.e., $P = 1$ in Eq. 48). For comparison of different scattering laws, the minimization was carried out using the Hapke’s law for macroscopically rough surface also, but the obtained parameters didn’t differ significantly (see Section 4.2).

The long-period component of the data together with the synthetic lightcurve of the best-fit solution are presented in Fig. 12. The region of plausible values of $\lambda_p$ and $\beta_p$ is plotted in Fig. 13 together with the best-fit solutions of all cited works. The solid angle that encloses this region is equal to 0.08 sr.
Figure 12: Long-period lightcurve component of 1996 FG$_3$. The observational data (points) are plotted with the synthetic lightcurve of the best-fit solution (curve). The lightcurve sequences from different dates are vertically offset by 0.1 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD$_0$) are listed in the left column for each curve.
Figure 13: Range of plausible poles of the mutual orbit of 1996 FG$_3$ in ecliptic coordinates. The best-fit solutions from this work (filled circle), from Mottola and Lahulla (2000; crosses), from Pravec et al. (2000; open circle) and from Scheirich (2003; open triangles) are denoted as well.
3.1.2 (88710) 2001 SL₉

The binary nature of this near-Earth asteroid was discovered by Pravec et al. (2006). They derived \( D_s/D_p = 0.28 \pm 0.02 \) and \( P_{\text{sym}}^{\text{orb}} = (16.40 \pm 0.02) \) h from their data. I reanalysed the data by Pravec et al. (2006) taken during 2001-10-10.4 to 10-20.9 using the model of two-axis ellipsoids on the eccentric orbit described in Section 2.4 and using their \( D_s/D_p \) and \( P_{\text{sym}}^{\text{orb}} \) as initial values. I found two solutions fitting the long-period component of the lightcurve satisfactorily. The parameters of these solutions are summarized in Table 4, with their errors obtained using procedure described in Section 2.5.2. The Lommel-Seeliger photometric law was used in the modeling and all lightcurve sequences of the data were taken as absolutely calibrated (i.e., \( P = 1 \) in Eq. 48).

The long-period component of the data together with the synthetic lightcurves of the best-fit solutions are presented in Fig. 14. The regions of plausible values of \( \lambda_p \) and \( \beta_p \) are plotted in Fig. 15 together with the best-fit solutions. The sum of solid angles enclosed by these regions is equal to 1.48 sr.

Actually, the positions of mutual orbit normal always lie on or close to the main circle perpendicular to the direction from the asteroid to the Sun or to the Earth. While the data of 2001 SL₉ don’t allow to resolve the position of the normal more precisely, the appearance of Fig. 15 is a logical result of this (since the asteroid was observed close to opposition, two main circles are almost indistinguishable).

3.1.3 (35107) 1991 VH

The binary nature of this near-Earth asteroid was discovered by Pravec et al. (1998). They derived \( P_{\text{sym}}^{\text{orb}} = 32.69 \pm 0.02 \) h, \( D_s/D_p = 0.40 \pm 0.02 \), \( a/D_p = 2.7 \pm 0.3 \) and \( \epsilon = 0.07 \pm 0.02 \) from data taken from February to April 1997.

Scheirich (2003) found two solutions from the 1997 data using spherical bodies on eccentric orbit: \( (\lambda_p = 330^\circ \pm 50^\circ, \beta_p = +75^\circ \pm 8^\circ) \), and \( (\lambda_p = 180^\circ \pm 30^\circ, \beta_p = -58^\circ \pm 9^\circ) \), with other estimated parameters consistent with that of Pravec et al. (1998) (these estimates are summarized in Pravec et al., 2006).

During the second apparition in February 2003, the asteroid was observed by Pravec et al. (2006). They constructed a numerical model consisting of two spherical bodies on a circular orbit and obtained four solutions. The estimated values of \( D_s/D_p = 0.37 \pm 0.03 \), \( a/D_p = 2.8 \pm 0.4 \) and \( \epsilon = 0.037^{+0.09}_{-0.027} \) were same for all four solutions, while the orbital pole coordinates and the sidereal period are summarized here (\( \lambda_p, \beta_p \) in degrees):

\[
\begin{align*}
I & : \quad \lambda_p = 241^{+7}_{-5}, \beta_p = +48^{+6}_{-37}, P_{\text{sym}}^{\text{orb}} = 32.61^{+0.07}_{-0.02} \text{ h} \\
II & : \quad \lambda_p = 190 \pm 20, \beta_p = -67^{+38}_{-9}, P_{\text{sym}}^{\text{orb}} = 32.64^{+0.07}_{-0.02} \text{ h} \\
III & : \quad \lambda_p = 56^{+16}_{-10}, \beta_p = +65^{+12}_{-32}, P_{\text{sym}}^{\text{orb}} = 32.57 \pm 0.05 \text{ h} \\
IV & : \quad \lambda_p = 38^{+4}_{-3}, \beta_p = -46^{+40}_{-8}, P_{\text{sym}}^{\text{orb}} = 32.58 \pm 0.02 \text{ h}
\end{align*}
\]

I reanalysed the data by Pravec et al. (1998) taken during 1997-02-27.0 to 04-04.9 and by Pravec et al. (2006) taken during 2003-02-01.8 to 02-29.0 using the
Figure 14: Long-period lightcurve component of (88710) 2001 SL₉. The observational data (points) are plotted with the synthetic lightcurves of the best-fit solutions (curves). The lightcurve sequences from different dates are vertically offset by 0.3 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD₀) are listed in the left column for each curve.
Figure 15: Range of plausible poles of the mutual orbit of (88710) 2001 SL₉ in ecliptic coordinates. The best-fit solutions from this work (filled circles) are denoted as well.
model of two-axis ellipsoids on the eccentric orbit described in Section 2.4. Five distinct solutions were found for 1997 apparition and four distinct solutions for 2003 apparition, fitting the long-period component of the lightcurve satisfactorily. The parameters of all solutions are summarized in Table 4, with their errors obtained using procedure described in Section 2.5.2. The Lommel-Seeliger photometric law was used in the modeling and all lightcurve sequences of the data were taken as absolutely calibrated (i.e., \( P = 1 \) in Eq. 48). For comparison of different scattering laws, the minimization was carried out using the Hapke’s law for macroscopically rough surface using 1997 apparition data also, but the obtained parameters didn’t differ significantly (see Section 4.2).

The long-period components of the data together with the synthetic lightcurves of the best-fit solutions are presented in Figs. 16 (1997 apparition) and 17 (2003 apparition). The regions of plausible values of \( \lambda_p \) and \( \beta_p \) are plotted in Figs. 18 (1997) and 19 (2003) together with the best-fit solutions of all cited works.

The solid angles that encloses the regions of 1997 and 2003 apparitions are equal to 0.21 sr and 0.35 sr, respectively.

Pravec et al. (2006) found three components with different periods in the 2003 data. In addition to 2.6236 h and 32.63 h periods, which were interpreted as primary’s rotational period and an orbital period of the secondary, respectively, another component with a period of 12.836 h was present. They suggest that the third component might be caused by a nonsynchronous rotation of the secondary or nonprincipal axis rotation of the primary. If the latter is the case, then a precession of the secondary’s orbit likely occur, which could be a natural explanation of the fact that pole solutions from the two different apparitions occupy disjoint regions.

If the rotation of the secondary is nonsynchronous, then the system is probably young and not tidally evolved. In that case an assumption that the rotational vector of the primary and the orbital vector of the secondary are not parallel could explain the precession of the secondary’s orbit too. While a vector about which the orbital pole of the secondary actually precesses is unknown, and due to ambiguities in the pole solution for both apparitions, only lower limit on the precession rate can be estimated. The closest two solutions of the orbital pole from the two apparitions are solution II. of 1997 and solution II. of 2003; they differ by approximately 22 degrees. The most extreme case of the precession axis to be perpendicular to the orbital pole gives a lower limit to the precession rate of 22 deg/6 yr \( \simeq 3.7 \) deg/yr. Only a very small flattening of the primary is required to explain this precession rate, as is shown in the next paragraph.

For the primary shape being a homogenous oblate ellipsoid and considering the effect of the oblateness up to the \( J_2 \) term (the first zonal harmonic coefficient in the gravitational potential expansion), the precession rate of nodes can be expressed as (Murray and Dermott, 1999, page 269)

\[
\dot{\Omega} = -\frac{3}{2} n_0 J_2 \left( \frac{A_p}{a} \right)^2 ,
\]

where \( n_0 \) is a mean motion of the secondary.\(^{16}\) The \( J_2 \) term can be expressed using moments of inertia of the primary with respect to the \( A_p, B_p, C_p \) axes, \( I_A, I_B, I_C \), as

\(^{16}\)Murray and Dermott use mean radius of primary \( R_p \) instead of its equatorial semiaxis \( A_p \), but
(see Heiskanen and Moritz, 2000, page 63)

\[ J_2 = \frac{I_C - \frac{I_A + I_B}{2}}{m_p A_p^2}, \]

where \( m_p \) is the mass of the primary. For homogenous oblate ellipsoid this expression could be modified as follows:

\[ J_2 = \frac{1}{5} - \frac{1}{5} \left( \frac{C_p}{A_p} \right)^2. \]

(61)

Combining the equation (61) with (60) and using the lower limit of the secondary’s orbit precession \( \dot{\Omega} = 3.7 \) deg/yr and other parameters of the system reported in Table 4, the lower limit on the primary’s flattening falls in the range of

\[ (A_p/C_p)_{\text{min}} \approx 1.0014 - 1.0025. \]

For the farthest two solutions of the orbital pole from the two apparitions (solutions III. of 1997 and II. of 2002), with angular distance about 170 degrees, the lower limit on the primary’s flattening obtained by the same process falls in the range of

\[ (A_p/C_p)_{\text{min}} \approx 1.0093 - 1.0202. \]

In other words, if the shape of the primary is only slightly different from sphere, any solution from the second apparition can be related to any solution from the first apparition.

### 3.1.4 (65803) Didymos

The binary nature of this near-Earth asteroid was discovered by Pravec et al. (2003b) and its lightcurve data were investigated by Pravec et al. (2006) in detail. They constructed a numerical model consisting of two spherical bodies on a circular orbit and obtained two solutions. The estimated parameters of these two solutions are summarized here (\( \lambda_p, \beta_p \) in degrees): I. \( P_{\text{orb}}^{\text{sid}} = 11.900 \pm 0.005 \) h, \( D_A/D_p = 0.22 \pm 0.02, \) \( a/D_p = 1.4 \pm 0.1, \) \( \epsilon = 0.06 \pm 0.04, \lambda_p = 156 \pm 2, \beta_p = +30 \pm 20, \) II. \( P_{\text{orb}}^{\text{sid}} = 11.920 \pm 0.005 \)

\[ \lambda_p = 330 \pm 20, \beta_p = -70 \pm 20. \]

I reanalysed the data by Pravec et al. (2006) taken during 2003-11-20.9 to 12-20.5 using the model of two-axis ellipsoids on the eccentric orbit described in Section 2.4. I found two solutions fitting the long-period component of the lightcurve satisfactorily. The parameters of these solutions, confirming the previous results, are summarized in Table 4, with their errors obtained using procedure described in Section 2.5.2. The Lommel-Seeliger photometric law was used in the modeling and all lightcurve sequences of the data were taken as absolutely calibrated (i.e., \( P = 1 \) in Eq. 48). For comparison of different scattering laws, the minimization was carried out using the Hapke’s law for macroscopically rough surface also, the obtained parameters are within 3\( \sigma \) errors of values derived using The Lommel-Seeliger law (see Section 4.2).

\[ \text{since this value only serves as a scaling factor, it could be replaced (in all expressions related to } J_2 \text{) by any other parameter.} \]
Figure 16: Long-period lightcurve component of (35107) 1991 VH (1997 apparition). The observational data (points) are plotted with the synthetic lightcurves of the best-fit solutions (curves). The lightcurve sequences from different dates are vertically offset by 0.3 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD$_0$) are listed in the left column for each curve.
Figure 17: Long-period lightcurve component of (35107) 1991 VH (2003 apparition). The observational data (points) are plotted with the synthetic lightcurves of the best-fit solutions (curves). The lightcurve sequences from different dates are vertically offset by 0.3 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD$_0$) are listed in the left column for each curve.
Figure 18: Range of plausible poles of the mutual orbit of 1991 VH (1997 apparition) in ecliptic coordinates. The best-fit solutions from this work (filled circles) and from Scheirich (2003; crosses) are denoted as well.
Figure 19: Range of plausible poles of the mutual orbit of 1991 VH (2003 apparition) in ecliptic coordinates. The best-fit solutions from this work (filled circles) and from Pravec et al. (2006; crosses) are denoted as well.
The long-period component of the data together with the synthetic lightcurves of the best-fit solutions are presented in Figs. 20 and 21. The regions of plausible values of \( \lambda_p \) and \( \beta_p \) are plotted in Fig. 22 together with the best-fit solutions from this work and from Pravec et al. (2006). The sum of solid angles enclosed by these regions is equal to 0.11 sr.

3.1.5 (3671) Dionysus

The binary nature of this near-Earth asteroid was discovered by Mottola et al. (1997) and its lightcurve data were investigated by Pravec et al. (2006) in detail. They constructed a numerical model consisting of two spherical bodies on a circular orbit and obtained two solutions. The estimated parameters of these two solutions are summarized here (\( \lambda_p, \beta_p \) in degrees): I. \( P_{\text{orb}}^{\text{sid}} = 27.72 \pm 0.02 \text{ h}, \ D_s/D_p = 0.20 \pm 0.02, \ a/D_p = 2.4^{+0.4}_{-0.2}, \ e = 0.07^{+0.03}_{-0.07}, \ \lambda_p = 311 \pm 7, \ \beta_p = +45 \pm 10, \ II. \ P_{\text{orb}}^{\text{sid}} = 27.74 \pm 0.02 \text{ h}, \ D_s/D_p = 0.20 \pm 0.02, \ a/D_p = 2.1^{+0.4}_{-0.3}, \ e = 0.09^{+0.04}_{-0.09}, \ \lambda_p = 57 \pm 20, \ \beta_p = +66^{+45}_{-20}.

I reanalysed the data by Pravec et al. (2006) taken during 1997-04-08.0 to 06-12.0 using the model of two-axis ellipsoids on the eccentric orbit described in Section 2.4. I found four solutions (two of them close to the previous results) fitting the long-period component of the lightcurve satisfactorily. The parameters of these solutions are summarized in Table 4, with their errors obtained using procedure described in Section 2.5.2. The Lommel-Seeliger photometric law was used in the modeling and all lightcurve sequences of the data were taken as absolutely calibrated (i.e., \( P = 1 \) in Eq. 48). For comparison of different scattering laws, the minimization was carried out using the Hapke’s law for macroscopically rough surface also, but the obtained parameters didn’t differ significantly (see Section 4.2).

The long-period component of the data together with the synthetic lightcurves of the best-fit solutions are presented in Fig. 23. The regions of plausible values of \( \lambda_p \) and \( \beta_p \) are plotted in Fig. 24 together with the best-fit solutions from this work and from Pravec et al. (2006). The sum of solid angles enclosed by these regions is equal to 0.31 sr.

3.1.6 (66391) 1999 KW4

The binary nature of this near-Earth asteroid was discovered by Benner et al. (2001) from radar data and by Pravec and Šarounová (2001) from photometry. The radar data were investigated by Ostro et al. (2006) and Scheeres et al. (2006). Ostro et al. constructed detailed shape models of both components and derive mutual orbit parameters as follows: \( P_{\text{orb}}^{\text{sid}} = 17.42 \pm 0.04 \text{ h}, \ \lambda_p = 326^\circ \pm 3^\circ, \ \beta_p = -62^\circ \pm 2^\circ, \ e = 0.0004 \pm 0.0019 \). While they used a general shapes of components, I converted characteristics of these shapes and the mutual orbit semimajor axis to the system defined in Sections 2.4.1 and 2.4.2 for easier comparison, approximating the equatorial radius of primary as \( A_p = \sqrt{(X_pY_p)/2} \), polar radius of primary as \( C_p = Z_p/2 \), polar radius of secondary as \( C_s = \sqrt{(Y_sZ_s)/2} \) and the longest semiaxis of the secondary as \( A_s = X_s/2 \), where \( X_p, Y_p, Z_p, X_s, Y_s, Z_s \) are extents along principal axes of the shape models of the primary and the secondary, respectively, taken from their Table 2.
Figure 20: Long-period lightcurve component (part I) of (65803) Didymos. The observational data (points) are plotted with the synthetic lightcurves of the best-fit solutions (curves). The lightcurve sequences from different dates are vertically offset by 0.2 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD₀) are listed in the left column for each curve. On the fourth curve from the top the minima are shown in an order opposite to other curves.
Figure 21: Long-period lightcurve component (part II) of (65803) Didymos. The observational data (points) are plotted with the synthetic lightcurves of the best-fit solutions (curves). The lightcurve sequences from different dates are vertically offset by 0.2 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD$_0$) are listed in the left column for each curve. On the second curve from the top the minima are shown in an order opposite to other curves.
Figure 22: Range of plausible poles of the mutual orbit of (65803) Didymos in ecliptic coordinates. The best-fit solutions from this work (open circles) and from Pravec et al. (2006; crosses) are denoted as well.
Figure 23: Long-period lightcurve component of (3671) Dionysus. The observational data (points) are plotted with the synthetic lightcurves of the best-fit solutions (curves). The lightcurve sequences from different dates are vertically offset by 0.2 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD₀) are listed in the left column for each curve.
Figure 24: Range of plausible poles of the mutual orbit of (3671) Dionysus in ecliptic coordinates. The best-fit solutions from this work (filled circles) and from Pravec et al. (2006; crosses) are denoted as well.
These converted parameters are:

\[
\begin{align*}
A_p/C_p &= 1.12 \pm 0.04, \\
A_s/C_s &= 1.4 \pm 0.1, \\
A_s/A_p &= 0.38 \pm 0.02, \\
a/A_p &= 3.37 \pm 0.07.
\end{align*}
\]

Pravec et al. (2006) analyzed photometric data obtained in June 2000 and from May to June 2001 and derived \( P_{\text{orb}}^{\text{max}} = 17.44 \pm 0.01 \) h and rough estimate of \( D_s/D_p \geq 0.3 \pm 0.01 \).

I reanalysed the data by Pravec et al. (2006) taken during 2001-05-25.0 to 06-20.9 using the model of two-axis ellipsoids on the eccentric orbit described in Section 2.4 and found single solution, close to the solution from radar, fitting the long-period component of the lightcurve satisfactorily. The parameters of this solution are summarized in Table 4, with their errors obtained using procedure described in Section 2.5.2. The Lommel-Seeliger photometric law was used in the modeling and all lightcurve sequences of the data were taken as absolutely calibrated (i.e., \( P = 1 \) in Eq. 48). For comparison of different scattering laws, the minimization was carried out using the Hapke's law for macroscopically rough surface also, but the obtained parameters didn't differ significantly (see Section 4.2).

The long-period component of 2001 data together with the synthetic lightcurve of the best-fit solution are presented in Fig. 26. The region of plausible values of \( \lambda_p \) and \( \beta_p \) is plotted in Fig. 27 together with the best-fit solution from 2001 photometric data and from Ostro et al. (2006). The solid angle enclosed by this region is equal to 0.06 sr. The region for 2000 apparition (see below) is shown there as well.

Due to insufficient quality and coverage of the 2000 data (taken during 2000-05-24.9 to 06-29.0, see Pravec et al., 2006, for details), it was not possible to run the full analysis of them and they were used for checking consistency with the model from 2001 data only. The shapes parameters of components and the orbit's semimajor axis were fixed at the best-fitting values from 2001 apparition, a circular orbit was assumed, and \( \lambda_p \) and \( \beta_p \) were stepped in a reasonable neighborhood around 2001 best-fit solution, while \( P_{\text{orb}}^{\text{max}} \) and \( L_0 \) were fitted. The region of plausible values of \( \lambda_p \) and \( \beta_p \) was then constructed using procedure described in Section 2.5.2 and is plotted in Fig. 27.

The pole solution derived by Ostro et al. (2006) fall within the region of \( \lambda_p \) and \( \beta_p \) constructed from the 2000 data (the long-period component of 2000 data together with the synthetic lightcurve constructed using the pole from Ostro et al. and the semimajor axis and shape parameters from the best-fit solution of 2001 photometric data are presented in Fig. 25.) but outside the region from 2001 data (see Fig. 27). The reason for this discrepancy is unknown. It can be caused by unmodeled features, such as secondary oscillation around center-to-center line, or by systematic errors in the lightcurve data.
Figure 25: Long-period lightcurve component of (66391) 1999 KW₄ (2000 apparition). The observational data (points) are plotted with the synthetic lightcurve (curve) constructed using the pole from Ostro et al. (2006) and the semimajor axis and shape parameters from the best-fit solution of 2001 photometric data. The lightcurve sequences from different dates are vertically offset by 0.5 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD₀) are listed in the left column for each curve.
Figure 26: Long-period lightcurve component of (66391) 1999 KW₄ (2001 apparition). The observational data (points) are plotted with the synthetic lightcurve of the best-fit solution (curve). The lightcurve sequences from different dates are vertically offset by 0.4 mag (first three curves from the top) and 0.3 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD₀) are listed in the left column for each curve.
Figure 27: Ranges of plausible poles of the mutual orbit of (66391) 1999 KW$_4$ for apparitions 2000 (dotted region) and 2001 (solid region) in ecliptic coordinates. The best-fit solutions from this work (apparition 2001; filled circle) and from Ostro et al. (2006; cross) are denoted as well.
3.1.7  2000 DP\textsubscript{107}

The binary nature of this near-Earth asteroid was discovered by Ostro et al. (2000) and Margot et al. (2000) from radar data and its lightcurve data were investigated by Pravec et al. (2006) in detail. Margot et al. (2002) analyzed the radar data obtained in September 2000 with the following results (\(\lambda_p, \beta_p\) in degrees): \(P_{\text{orb}}^{\text{sid}} = 42.12 \pm 0.05\) h, \(D_s/D_p = 0.4 \pm 0.2\), \(a/D_p = 3.3 \pm 0.7\), \(e = 0.010 \pm 0.005\), \(\lambda_p = 280 \pm 27\) and \(\beta_p = +73 \pm 7\).

Pravec et al. (2006) estimated synodic orbital period as \(P_{\text{orb}}^{\text{syn}} = 42.2 \pm 0.1\) h. I reanalysed their data taken during 2000-09-25.2 to 10-05.2 using the model of two-axis ellipsoids on the eccentric orbit described in Section 2.4. I found two solutions fitting the long-period component of the lightcurve satisfactorily. The second solution agrees with the radar solution within the error bars. The parameters of these solutions are summarized in Table 4, with their errors obtained using procedure described in Section 2.5.2. The Lommel-Seeliger photometric law was used in the modeling and all lightcurve sequences of the data were taken as absolutely calibrated (i.e., \(P = 1\) in Eq. 48).

The long-period component of the data together with the synthetic lightcurves of the best-fit solutions are presented in Fig. 28. The regions of plausible values of \(\lambda_p\) and \(\beta_p\) are plotted in Fig. 29 together with the best-fit solutions from this work and from Margot et al. (2002). The sum of solid angles enclosed by these regions is equal to 0.16 sr.

3.1.8  (66063) 1998 RO\textsubscript{1}

The binary nature of this near-Earth asteroid was discovered from photometric data and confirmed by radar by Pravec et al. (2003a). Pravec et al. (2006) investigated its lightcurve data taken in four apparitions – in September 2001, 2002, 2003 and 2004. They derived \(P_{\text{orb}}^{\text{syn}} = 14.53 \pm 0.02\) h and 14.55 \(\pm 0.01\) h from the 2003 and 2004 data, respectively. The rough estimate of the orbital period from the less abundant data of 2002 was \(P_{\text{orb}}^{\text{syn}} = 14.45 \pm 0.05\). In the 2001 data the secondary component wasn’t resolved due to very short time span of data (3.5 h). From the depth of the secondary event detected in 2002 they estimated \(D_s/D_p = 0.48 \pm 0.03\).

I reanalysed the data by Pravec et al. (2006) taken during 2003-09-16.8 to 09-25.9 and during 2004-09-10.2 to 09-21.3 using the model of two-axis ellipsoids on the eccentric orbit described in Section 2.4 and found two solutions for 2003 apparition and single solution for 2004, fitting the long-period component of the lightcurve satisfactorily. The parameters of this solutions are summarized in Table 4, with their errors obtained using procedure described in Section 2.5.2. The Lommel-Seeliger photometric law was used in the modeling and all lightcurve sequences of the data were taken as absolutely calibrated (i.e., \(P = 1\) in Eq. 48). For comparison of different scattering laws, the minimization was carried out using the Hapke’s law for macroscopically rough surface for 2004 data also, but the obtained parameters didn’t differ significantly (see Section 4.2).

The long-period components of 2003 and 2004 data together with the synthetic lightcurves of the best-fit solutions are presented in Figs. 31 and 32, respectively. The regions of plausible values of \(\lambda_p\) and \(\beta_p\) are plotted in Figs. 33 (2003) and 34.
Figure 28: Long-period lightcurve component of 2000 DP$_{107}$. The observational data (points) are plotted with the synthetic lightcurves of the best-fit solutions (curves). The lightcurve sequences from different dates are vertically offset by 0.3 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD$_0$) are listed in the left column for each curve.
Figure 29: Range of plausible poles of the mutual orbit of 2000 DP$_{107}$ in ecliptic coordinates. The best-fit solutions from this work (open circles) and from Margot et al. (2002; cross) are denoted as well.
Figure 30: Composite long-period lightcurve component of (66063) 1998 RO\textsubscript{1} (2002 apparition). The observational data (points) are plotted with the synthetic lightcurves (curves) constructed using the parameters of best-fit solutions from 2003 and 2004 apparition.

(2004) together with the best-fit solutions from this work. The sums of solid angles enclosed by the regions for 2003 and 2004 data are equal to 0.35 sr and 0.08 sr, respectively.

Due to insufficient coverage of the 2002 data (taken during 2002-09-13.0 to 09-16.0, see Pravec et al., 2006, for details), it was not possible to run the full analysis of them and they were used for checking consistency with the models from 2003 and 2004 data only. Fig. 30 shows the long-period component of the data together with the synthetic lightcurves of the best-fit solutions from 2003 and 2004 apparition (the secondary’s argument of mean length \( L_0 \) was fitted to match the phase of the events).

The first solution of the 2003 apparition (\( \lambda_p \sim 260^\circ, \beta_p \sim 60^\circ \)) gives a better fit to the observed data and, based on the similar pole position of the single solution of the 2004 apparition, it is probably the only real solution. As can be seen in Fig. 30, the second solution of the 2003 apparition gives also the worst fit to the 2002 data, which supports the above hypothesis.
Figure 31: Long-period lightcurve component of (66063) 1998 RO₁ (2003 apparition). The observational data (points) are plotted with the synthetic lightcurves of the best-fit solutions (curve). The lightcurve sequences from different dates are vertically offset by 0.4 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD₀) are listed in the left column for each curve. On the third and sixth curve from the top the minima are shown in an order opposite to other curves.
Figure 32: Long-period lightcurve component of (66063) 1998 RO₁ (2004 apparition). The observational data (points) are plotted with the synthetic lightcurve of the best-fit solution (curve). The lightcurve sequences from different dates are vertically offset by 0.3 mag for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD₀) are listed in the left column for each curve. On the first, second, sixth and tenth curve from the top the minima are shown in an order opposite to other curves.
Figure 33: Ranges of plausible poles of the mutual orbit of (66063) 1998 RO₁ for apparition 2003 in ecliptic coordinates. The best-fit solutions (filled circles) are denoted as well.
Figure 34: Range of plausible poles of the mutual orbit of (66063) 1998 RO₁ for apparition 2004 in ecliptic coordinates. The best-fit solutions (filled circles) are denoted as well.
3.2 Dependence of modeled parameters on observational circumstances

A natural question about orbital poles of the modeled binary system arises: what is the dependence of the number of solutions and the sum of solid angles enclosed by the regions of plausible orbital poles (called SA hereafter) on the geometry of observations? In principle, the longer the sky arc spanned by an asteroid during the observation, the smaller SA should be expected. The observational circumstances of the modeled systems are summarized in Table 3. The number of solutions and SA are plotted vs. the sky arc in panels a and b of Fig. 35 for all these systems. If only occultations or eclipses are present in the lightcurves of all solutions for particular system, then the arc spanned by the asteroid with respect to the Earth or to the Sun, respectively, has to be used in this plot. But since in all cases there are the solutions with both types of events (see Table 5), the longest of the two arcs is used.

From the panels a and b of Fig. 35 the trend of increasing number of solutions and SA with decreasing sky arc is recognizable, though it is obvious that other effects, such as number of lightcurve sequences within the observed arc, their cadence, and the quality of the data, affect the results as well. This is well demonstrated in the case of (88710) 2001 SL₉ which, in spite of the fact it hasn’t the shortest arc of observation, it has the largest SA due to insufficient coverage and noisy data.

The evolution of lengths and shapes of the mutual events in the lightcurve with the changing geometry is also crucial for a restriction of the primary’s flattening (expressed as $A_p/C_p$, see Section 2.4.1). This is demonstrated in panel c of Fig. 35, where a similar chart to the other two is plotted, but instead of number of solution or SA, the largest reasonable flattening of the primary (over all solutions for particular asteroid) is displayed. For very short arcs large values of the primary’s flattening are fitting the observed data equally well and the flattening can be easily overestimated. While the uncertainty in the primary’s shape is a large source of error in the bulk density of the system, this overestimation is applied on the density as well, as is shown in panel d of Fig. 35. Analogous to the panel c, the largest reasonable values of bulk densities are plotted there.

Although not observed in any of the modeled lightcurves, a phenomenon important for the reduction of number of solutions, SA, and maximal flattening of primary, is also a disappearance (or reappearance) of the mutual events in the course of observation. For that reason, the sky arcs of observed binaries have to be extended as much as possible.
Figure 35: Dependence of parameters of the modeled systems on a sky arc spanned by an asteroid during the observation. See text for details.
4 Discussion

4.1 Single body lightcurve inversion and shape of primaries

This subsection is aimed to review some basics of a technique developed and applied by M. Kaasalainen et al. for single body lightcurve inversion in recent years, to compare the technique with the method presented in this work, and to test its applicability to primaries of observed binaries.

For convex representation of the shape of the body, a convenient method for shape inversion was developed by Kaasalainen and Torppa (2001, see also Kaasalainen et al., 2002). The total brightness of a convex shape is a function of the sum $\sum A_i r_i \cos \epsilon_i$, where $A_i$ are areas of the facets, $r_i$ are their bidirectional reflectances defined by scattering law (Eq. 38), and $\epsilon_i$ are angles of emergence of the facets. This sum is not dependent on actual shapes and radius vectors of individual facets, but only on the areas of the facets $A_i$ and their orientations defined by normals to the surface of the facets, $\mathbf{n}_i$. The normals $\mathbf{n}_i$ can be chosen a priori and hold fixed. Therefore, in the simplest case when the pole direction and the rotational period are known, the only parameters that have to be fitted during the optimization are areas of the facets $A_i$ corresponding to their normals $\mathbf{n}_i$. Once the areas of the facets are known, they are inverted into the radius vectors of the vertices using Minkowski minimization (Kaasalainen and Torppa, 2001).

For nonconvex shapes (as well as for binaries as a special case) the mutual occultations and shadowing between the facets take place. These effects (represented by the visibility factors $o_i$ in Eq. 37) are dependent on the actual shape of the body and therefore the radius vectors of the facets have to be fitted during the optimization as well.

Kaasalainen et al. (2002b) stated that three or four apparitions are usually required for modeling the shapes of the asteroids, while one or two apparitions are sufficient only if the observing geometry varied considerably during them. From the binaries listed in Table 3, none of them satisfy these requirements. The geometries of multi-apparition data of (66063) 1998 RO$_1$ and (35107) 1991 VH were almost the same for all apparitions. In case of (66391) 1999 KW$_4$, the change of the geometry was substantial during the 2001 apparition, but the quality of the data was insufficient to obtain the shape of the primary from the photometry.

For the purpose of testing the applicability, the short-period component of the lightcurve of (65803) Didymos taken during 2003-11-20.9 to 12-20.5 was used by J. Durech (personal communication) to create a shape model and the rotational state of the primary using the methods developed by Kaasalainen et al. The best-fit sidereal rotational period is 2.25959 h and ecliptic coordinates of the pole of rotation are $\lambda_{prim} = 155^\circ$, $\beta_{prim} = 41^\circ$, which is well within the range of plausible pole of the orbit of the secondary (see Fig. 22). The obtained shape solution is presented in Fig. 36. The short-period component together with the synthetic lightcurve generated using the obtained shape are presented in Fig. 37. The presented shape model of (65803) should not be considered as unique and definite. As could be seen in Fig. 37, it does not even fit the data in the last five sequences well.

The knowledge of the actual shape of primary would be useful to better constrain
the bulk density of the system. If the data do not cover sufficient range of geometries and the unique shape model could not be obtained, a set of shapes giving satisfactory fits to the observed primary’s rotational curve should be created in order to derive a range of plausible densities. This can be done by a number of runs of shape inversion started at different initial shapes, e.g., Gaussian random spheres (Muinonen and Lagerros, 1998).

4.2 Dependence of fitted parameters on scattering law

As noted in Section 3.1, the Hapke’s law for macroscopically rough surface was also tested for some selected binaries for comparison. The same minimization procedure was used as in the case with the Lommel-Seeliger law. Spectroscopic classes of these asteroids, values of the Hapke’s parameters derived for these classes by other researches and the corresponding references are given in Table 6. In all cases, the cited values of Hapke’s parameters were derived from photometric data for asteroids other than modeled in this work, but with the same or similar spectral classes.

The parameters of all best-fit solutions found using the Hapke’s law lie within the 3-σ error bars of values obtained using L-S law; most of them differ only insignificantly. Comparison of results obtained with the two laws is briefly summarized in Table 7. The 3-σ errors of selected parameters from Table 4 (odd lines) are presented together with the differences $\Delta_{HR}(p) = (p)_{HR} - (p)_{LS}$ (even lines), where $(p)_{HR}$ and $(p)_{LS}$ are the best-fit values of some parameter $p$ obtained using the Hapke’s law and the Lommel-Seeliger law, respectively. Only the values of the best-fit solutions for each binary are shown.

4.3 Effect of finite number of facets

The approximation of the components’ shapes by finite number of facets introduces deviations of the synthetic lightcurves from the ideal shape; the largest deviations occur during the mutual events. Assessment of these deviations was done by comparing the lightcurves synthesized using models with different number of facets. The “real” lightcurves were realized using spheres approximated by 101 860 and 25 460
Figure 37: Short-period lightcurve component of (65803) Didymos. The observational data (points) are plotted with the synthetic lightcurve (curve) generated using the shape model presented in Fig. 36. The lightcurve sequences from different dates are vertically offset for clarity, and different symbols are used for them to avoid an overlap or confusion. The offsets in date (JD₀) are listed in the left column for each curve.

Table 6: Spectral classes of systems modeled with Hapke’s law for macroscopically rough surface, and values of parameters used.

<table>
<thead>
<tr>
<th>Object</th>
<th>Sp. class</th>
<th>r</th>
<th>B₀</th>
<th>h</th>
<th>g</th>
<th>θ(°)</th>
<th>Hapke parameters and reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996 FG₃</td>
<td>C (Burbine, 2000)</td>
<td>0.048</td>
<td>1.6</td>
<td>0.060</td>
<td>0.40</td>
<td>5</td>
<td>Bowell et al. (1989)</td>
</tr>
<tr>
<td>(35107) Sk</td>
<td>(Binzel et al., 2004)</td>
<td>0.173</td>
<td>1.03</td>
<td>0.024</td>
<td>-0.34</td>
<td>20</td>
<td>Hudson et al. (2000)</td>
</tr>
<tr>
<td>(65803) M</td>
<td>(Pravec et al., 2006)</td>
<td>0.154</td>
<td>0.94</td>
<td>0.036</td>
<td>0.40</td>
<td>35</td>
<td>Bowell et al. (1989)</td>
</tr>
<tr>
<td>(36711) Cb</td>
<td>(Bus and Binzel, 2002)</td>
<td>0.204</td>
<td>0.47</td>
<td>0.030</td>
<td>0.60</td>
<td>25</td>
<td>Bowell et al. (1989)</td>
</tr>
<tr>
<td>(66381) S</td>
<td>(Binzel et al., 2004)</td>
<td>0.173</td>
<td>1.03</td>
<td>0.024</td>
<td>-0.34</td>
<td>20</td>
<td>Hudson et al. (2000)</td>
</tr>
<tr>
<td>(66063) S</td>
<td>(Abell et al., 2005)</td>
<td>0.173</td>
<td>1.03</td>
<td>0.024</td>
<td>-0.34</td>
<td>20</td>
<td>Hudson et al. (2000)</td>
</tr>
</tbody>
</table>
Table 7: 3-σ errors of selected parameters from Table 4 and the differences between the values of these parameters obtained using Lommel-Seeliger law and Hapke’s law for macroscopically rough surface.

<table>
<thead>
<tr>
<th>Object (apparition)</th>
<th>3-σ($A_s/A_p$)</th>
<th>3-σ($a/A_p$)</th>
<th>3-σ($\lambda_p$)</th>
<th>3-σ($\beta_p$)</th>
<th>3-σ($P^{sid}_{ord}$)</th>
<th>3-σ($A_p/C_p$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996 FG$_2$ (1998)</td>
<td>+0.07; -0.08</td>
<td>+0.2; -0.5</td>
<td>±0.6</td>
<td>±14; -5</td>
<td>±0.01</td>
<td>±0.5; -0.2</td>
</tr>
<tr>
<td>(35107) (1997)</td>
<td>-0.01</td>
<td>-0.03</td>
<td>-1</td>
<td>+0.4</td>
<td>+0.0002</td>
<td>+0.4; -0.5</td>
</tr>
<tr>
<td>(65803) (2003)</td>
<td>+0.09; -0.10</td>
<td>+0.3; -0.6</td>
<td>+6; -26</td>
<td>+7; -18</td>
<td>±0.01</td>
<td>±0.4; -0.5</td>
</tr>
<tr>
<td>(3671) (1997)</td>
<td>±0.02</td>
<td>±0.2</td>
<td>±4; -7</td>
<td>±45; -15</td>
<td>+0.004; -0.01</td>
<td>+0.3; -0.0</td>
</tr>
<tr>
<td>(66391) (2001)</td>
<td>-0.05</td>
<td>-0.09</td>
<td>±0.8</td>
<td>±3</td>
<td>-0.006</td>
<td>O</td>
</tr>
<tr>
<td>(66063) (2004)</td>
<td>+0.08; -0.16</td>
<td>+0.6; -1.0</td>
<td>+26; -18</td>
<td>+14; -13</td>
<td>+0.03; -0.02</td>
<td>+0.6; -0.5</td>
</tr>
<tr>
<td></td>
<td>-0.01</td>
<td>-0.04</td>
<td>±3</td>
<td>-6</td>
<td>+0.01</td>
<td>+0.5</td>
</tr>
<tr>
<td></td>
<td>±0.02</td>
<td>±0.2</td>
<td>±0.6</td>
<td>±26; -64</td>
<td>±17; -11</td>
<td>±0.02</td>
</tr>
<tr>
<td></td>
<td>±0.003</td>
<td>±0.06</td>
<td>±10</td>
<td>-3</td>
<td>-0.001</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

Notes. $\lambda_p$, $\beta_p$ in degrees, $P^{sid}_{ord}$ in hours. See text for details.

facets for the primary and the secondary, respectively, and the lightcurves constructed using lower number of facets were compared with them. In order to cover various geometries, these comparisons were done for the majority of the modeled systems. Orbital parameters of the best solutions for spherical shapes and eccentric orbits were used. The maximal residuals of tested lightcurves with respect to the "real" ones are summarized in Table 8, as well as their root mean square values (RMS). Since outside the mutual events the synthetic curves are constant and don’t differ for spherical bodies, the RMS are computed from the points inside the events only. Since the residuals aren’t systematic and on long timescales the effects of their positive and negative values rather cancel out, they don’t affect the fitted parameters of the systems significantly. For verification of this assumption, data of two binaries with the largest deviations obtained for the smallest number of facets, 2000 DP$_{107}$ and 1998 RO$_1$ (apparition 2004), were fitted using higher number of facets (6368 and 1584 for the primary and the secondary) also. The best-fit parameters obtained were within 10% (for 2000 DP$_{107}$) and 20% (for 1998 RO$_1$) of their error bars presented in Table 4.

4.4 Effect of synthetic curve interpolation

The process of interpolation of the synthetic lightcurves described in Section 2.4.5 is another source of deviations from ideal shape of the curve. The interpolation interval was set to 1/100 of day for all modeled systems. The maximal residuals of interpolated lightcurves with respect to the uninterpolated ones (having all other parameters unchanged) are summarized in Table 9, as well as their root mean square values (RMS). Orbital and shape parameters of the best solutions from Table 4 were used. The RMS of the residuals are smaller than 0.007 mag for all cases and don’t affect the fitted parameters of the systems significantly. The data of the system with the largest RMS, 1998 RO$_1$, were fitted in both apparitions using uninterpolated synthetic lightcurves also, and obtained best-fit parameters were within 20% of their error bars presented in Table 4.
Table 8: Residuals between synthetic lightcurves generated using shapes approximated with various number of facets.

<table>
<thead>
<tr>
<th>Object (apparition)</th>
<th>( N_1 )</th>
<th>( N_2 )</th>
<th>Max. resid.</th>
<th>Min. resid.</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>101860</td>
<td>25460</td>
<td>25460</td>
<td>6368</td>
<td>6368</td>
</tr>
<tr>
<td>1996 FG9 (1998)</td>
<td></td>
<td></td>
<td>+0.00019</td>
<td>+0.00063</td>
<td>+0.00435</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00023</td>
<td>-0.00054</td>
<td>-0.00166</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00006</td>
<td>0.00021</td>
<td>0.00130</td>
</tr>
<tr>
<td>(88710) (2001)</td>
<td></td>
<td></td>
<td>+0.00022</td>
<td>+0.00063</td>
<td>+0.00361</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00012</td>
<td>-0.00024</td>
<td>-0.00156</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00006</td>
<td>0.00021</td>
<td>0.00139</td>
</tr>
<tr>
<td>(35107) (1997)</td>
<td></td>
<td></td>
<td>+0.00026</td>
<td>+0.00064</td>
<td>+0.00286</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00014</td>
<td>-0.00035</td>
<td>-0.00237</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00006</td>
<td>0.00023</td>
<td>0.00140</td>
</tr>
<tr>
<td>(35107) (2003)</td>
<td></td>
<td></td>
<td>+0.00021</td>
<td>+0.00084</td>
<td>+0.00485</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00018</td>
<td>-0.00043</td>
<td>-0.00975</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00007</td>
<td>0.00027</td>
<td>0.00219</td>
</tr>
<tr>
<td>(65803) (2003)</td>
<td></td>
<td></td>
<td>+0.00028</td>
<td>+0.00114</td>
<td>+0.00708</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00016</td>
<td>-0.00042</td>
<td>-0.00117</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00006</td>
<td>0.00024</td>
<td>0.00149</td>
</tr>
<tr>
<td>(3671) (1997)</td>
<td></td>
<td></td>
<td>+0.00023</td>
<td>+0.00114</td>
<td>+0.00528</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00028</td>
<td>-0.00120</td>
<td>-0.00528</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00007</td>
<td>0.00027</td>
<td>0.00339</td>
</tr>
<tr>
<td>(66391) (2001)</td>
<td></td>
<td></td>
<td>+0.00032</td>
<td>+0.00117</td>
<td>+0.00711</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00029</td>
<td>-0.00084</td>
<td>-0.00425</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00009</td>
<td>0.00035</td>
<td>0.00186</td>
</tr>
<tr>
<td>2000 DP107 (2000)</td>
<td></td>
<td></td>
<td>+0.00023</td>
<td>+0.00063</td>
<td>+0.001269</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00016</td>
<td>-0.00065</td>
<td>-0.00572</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00007</td>
<td>0.00032</td>
<td>0.00396</td>
</tr>
<tr>
<td>(66063) (2003)</td>
<td></td>
<td></td>
<td>+0.00036</td>
<td>+0.00083</td>
<td>+0.00860</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00023</td>
<td>-0.00066</td>
<td>-0.00530</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00008</td>
<td>0.00034</td>
<td>0.00290</td>
</tr>
<tr>
<td>(66063) (2004)</td>
<td></td>
<td></td>
<td>+0.00048</td>
<td>+0.00151</td>
<td>+0.01219</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.00036</td>
<td>-0.00111</td>
<td>-0.00575</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.00009</td>
<td>0.00035</td>
<td>0.00307</td>
</tr>
</tbody>
</table>

Notes. \( N_1 \) and \( N_2 \) are number of facets for the primary and the secondary, respectively. The shapes of bodies are taken as spheres. See text for other details.
Table 9: Residuals between interpolated and uninterpolated synthetic lightcurves.

<table>
<thead>
<tr>
<th>Object (apparition)</th>
<th>Max. resid.</th>
<th>Min. resid.</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996 FG₃ (1998)</td>
<td>+0.0090</td>
<td>−0.0168</td>
<td>0.0023</td>
</tr>
<tr>
<td>(88710) (2001)</td>
<td>+0.0074</td>
<td>−0.0055</td>
<td>0.0011</td>
</tr>
<tr>
<td>(35107) (1997)</td>
<td>+0.0070</td>
<td>−0.0069</td>
<td>0.0012</td>
</tr>
<tr>
<td>(35107) (2003)</td>
<td>+0.0083</td>
<td>−0.0095</td>
<td>0.0011</td>
</tr>
<tr>
<td>(65803) (2003)</td>
<td>+0.0061</td>
<td>−0.0078</td>
<td>0.0010</td>
</tr>
<tr>
<td>(3671) (1997)</td>
<td>+0.0092</td>
<td>−0.0092</td>
<td>0.0012</td>
</tr>
<tr>
<td>(66391) (2001)</td>
<td>+0.0121</td>
<td>−0.0231</td>
<td>0.0034</td>
</tr>
<tr>
<td>2000 DP₁₀₇ (2000)</td>
<td>+0.0138</td>
<td>−0.0134</td>
<td>0.0036</td>
</tr>
<tr>
<td>(66063) (2003)</td>
<td>+0.0128</td>
<td>−0.0300</td>
<td>0.0064</td>
</tr>
<tr>
<td>(66063) (2004)</td>
<td>+0.0099</td>
<td>−0.0155</td>
<td>0.0048</td>
</tr>
</tbody>
</table>

Notes. See text for details.

5 Conclusions and future work

I reviewed the basic properties of binary asteroids populations, their evolution and stability, and previous work on modeling of their lightcurves.

The numerical model for inversion of lightcurves of binary asteroids has been developed and described. The semianalytic algorithm for precomputing some of the parameters of the mutual orbit was developed as well, which, together with presented interpolation of synthetic lightcurve, increased the speed of inversion process by more than an order of magnitude.

Eight binary near-Earth asteroid were modeled using the presented methods, some of them in multiple apparitions.

A special emphasis was given to assess the errors of parameters. This was achieved by detailed examination of synthetic lightcurves generated using parameters varied around plausible solutions. Formal χ²-error estimates would lead to much smaller and unrealistic uncertainties due to the model idealizations and systematic errors.

More realistic shapes of the system's components were used compared to spherical shapes used in the previous work (Pravec et al. 2006). The results led to the conclusion that it is impossible to derive the flattening of the primaries, and therefore the bulk density of the systems also, without large errors bars, if the systems are observed at short sky arcs. Therefore, caution should be taken in interpreting low values of the bulk densities, around ~ 1 g cm⁻³, presented by Pravec et al. (2006) or Behrend et al. (2006). By increasing the sky arc the flattening of primaries and the bulk density could be better constrained. The observers are therefore encouraged to extend the sky arcs of observed binaries as much as possible.

As shown on five modeled systems, 1996 FG₃, (65803), (66391), 2000 DP₁₀₇ and (66063), the poles of the mutual orbits can be estimated with quite small uncertainties, although the poles are not unambiguous in some cases. Future analysis of larger sample of binaries¹⁷ may allow a debiased distribution of the mutual poles to be constructed that could be used to test theories of mutual orbits evolution. Reflection and reemission of sunlight affecting spin states of single asteroids (the

YORP effect) can drive their obliquities toward asymptotic values. The YORP effect combined with planetary perturbation also provide plausible mechanism to explain non-Maxwellian distribution of obliquities of Koronis family members (Bottke et al. 2006, and references therein). As shown by Pravec et al. (2002), the whole population of single asteroids exhibits a bimodality in spin-vector distribution. Though Čuk and Burns (2005) shown that the YORP effect affect the mutual orbits of binaries significantly, their results regarding expected pole distribution are very preliminary. None similar study has been carried out for binaries so far, neither observational nor theoretical. It is therefore highly desirable to create a debiased pole distribution for larger sample of binaries.

If the pole of the mutual orbit and the spin axis of the primary are not parallel, then the mutual orbit may precess in general. While the observed data do not allow to derive the spin axes of primaries, they are assumed to be normal to the mutual orbits in the analysis. Three modeled systems were observed during several apparitions; the result for two of them, (66391) 1999 KW$_4$ and (66063) 1998 RO$_1$, indicated that this was the right assumption, because no change in the orbital pole was detected within the uncertainties. In the case of (35107) 1991 VH, the orbital poles cannot be determined unambiguously for neither of both apparitions. However, the pole solutions from the two apparitions occupy disjoint regions, which can be interpreted as the pole precession. The lower limit of this precession is $\simeq 3.7$ deg/yr. Since only very small flattening of the primary ($\leq 1.02$) is required to link any two solutions from both apparitions to each other by precession of the orbital pole, the precession seems to be the simplest explanation.

Future work will be directed to three main topics:

- Increase the number of modeled binaries and extend the results to small Main Belt binaries.

- Develop a method for inversion of multi-apparition data simultaneously, including an effect of precession of the mutual orbit.

- Simulate the selection effects in the observed mutual orbits’ poles and construct a debiased distribution of the poles for NEA and Main Belt binaries.
6 Appendix

6.1 Explanation of symbols used in text

The symbols used for a specific purpose in a single section only are omitted.

- \( a \) Semimajor axis of the mutual orbit
- \( A_p, C_p \) Equatorial and polar semiaxes of the primary represented as an oblate spheroid
- \( A_s, C_s \) The longest and the shortest semiaxes of the secondary represented as a prolate spheroid
- \( B \) Vector of fitted parameters
- \( B_0 \) Amplitude of the opposition effect in Hapke’s photometric law
- \( c \) Unit vector heading from primary’s to secondary’s center
- \( C_0, C_1 \) Mean reduced light fluxes of observed lightcurve and of its short-period component
- \( D_p, D_s \) Mean projected diameters of the primary and the secondary
- \( e \) Eccentricity of the mutual orbit
- \( g \) Asymmetry factor of the phase function in Hapke’s photometric law
- \( h \) Angular width of the opposition effect in Hapke’s photometric law
- \( L \) \((- \omega + M)\) Argument of mean length of the secondary
- \( L' \) Time-increasing value of \( L \) (not restricted to interval \([0, 2\pi]\) only)
- \( L_0 \) Argument of mean length of the secondary for a given epoch \( T_0 \)
- \( M \) Mean anomaly of the secondary
- \( n \) Mean motion of the secondary
- \( n \) Unit vector perpendicular to the secondary’s orbit
- \( P_1, P_2 \) Periods of components of the lightcurve (the nature of components is not specified)
- \( P_{\text{syn}}, P_{\text{sid}} \) Synodic and sidereal period of the secondary
- \( P_{\text{prim}} \) Rotational period of the primary
- \( R_p, R_s \) \( D_p/2, D_s/2 \)
- \( s, h \) Unit vectors heading from the asteroid to the Sun and to the Earth
- \( T \) Time epoch
- \( v \) True anomaly of the secondary
- \( \alpha \) Phase angle
- \( \beta_s \) Angle between \( s \) (or \( h \)) and \( n \)
- \( \delta_1 \) Angle between \( c \) and \( s \) (or \( h \)) for the first and the fourth contact
- \( \delta_2 \) Angle between \( c \) and \( s \) (or \( h \)) for the second and the third contact
- \( \lambda_p, \beta_p \) Pole coordinates of the mutual orbit in ecliptic frame
- \( \theta \) Mean slope angle of surface facets in Hapke’s photometric law
- \( \rho \) Bulk density of binary system
- \( \omega \) Argument of pericenter of the mutual orbit

6.2 Solving of equations for finding contacts

Solving the system of equations (see Section 2.2.1)

\[
\begin{align*}
\mathbf{c} \cdot \mathbf{s} &= \cos \delta, \\
\mathbf{c} \cdot \mathbf{n} &= 0, \\
|\mathbf{c}| &= 1,
\end{align*}
\]

(62) (63) (64)
for \( \mathbf{c} \):

At first, we define two vectors

\[
\mathbf{a} = \mathbf{n} \times \mathbf{s},
\]

\[
\mathbf{b} = \mathbf{a} \times (\cos \delta \mathbf{n}).
\]

Using vector formula

\[
\mathbf{c} \times (\mathbf{n} \times \mathbf{s}) = \mathbf{n}(\mathbf{c} \cdot \mathbf{s}) - \mathbf{s}(\mathbf{c} \cdot \mathbf{n})
\]

and equations (62), (63) and (65) we get

\[
\mathbf{c} \times \mathbf{a} = \cos \delta \mathbf{n}.
\]

Multiplying this equation with \( \mathbf{a} \times \) and using (66) we get

\[
\mathbf{a} \times (\mathbf{c} \times \mathbf{a}) = \mathbf{b}.
\]

This could be modified to

\[
\mathbf{c}(\mathbf{a} \cdot \mathbf{a}) - \mathbf{a}(\mathbf{a} \cdot \mathbf{c}) = \mathbf{b}.
\]

Defining

\[
\alpha = \mathbf{a} \cdot \mathbf{c}
\]

we get solution for \( \mathbf{c} \):

\[
\mathbf{c} = \frac{\alpha \mathbf{a} + \mathbf{b}}{a^2}
\]

with unknown parameter \( \alpha \). This parameter could be found using (64):

\[
1 = \mathbf{c} \cdot \mathbf{c} = \frac{1}{a^2}(\alpha \mathbf{a} + \mathbf{b}) \cdot \frac{1}{a^2}(\alpha \mathbf{a} + \mathbf{b})
\]

and thus

\[
a^4 = \alpha^2 a^2 + b^2 + 2\alpha \mathbf{a} \cdot \mathbf{b}.
\]

The last term is zero since \( \mathbf{a} \perp \mathbf{b} \), and thus

\[
\alpha = \pm \sqrt{a^2 - \frac{b^2}{a^2}},
\]

where \( a = |\mathbf{a}| \) and \( b = |\mathbf{b}| \).
7 Publications and relevant citations

7.1 Refereed papers


10. Shepard, Michael K.; Margot, Jean-Luc; Magri, Christopher; Nolan, Michael C.; Schlieder, Joshua; Estes, Benjamin; Bus, Schelte J.; Volquardsen, Eric L.; Rivkin, Andrew S.; Benner, Lance A. M.; Giorgini, Jon D.; Ostro, Steven J.; Busch, Michael W.: Icarus 184, 198-210, 2006.


7.2 Posters and talks


Scheirich P. Tumbling asteroids models. IAU Symposium No. 229, Asteroids, Comets, Meteors. Búzios, Rio de Janeiro, Brazil, 2005 (poster)


8 References


Arlot, J. E., Lecacheux, J., Richardson, Ch., Thuillot, W., 1985. A possible satellite of (146) Lucina. Icarus 61, 224


Belton, M., Carlson, R., 1994. 1993 (243) 1. IAU Circ. 5948


86


88


Heiskanen, W. A., Moritz, H., 2000. Physical Geodesy. Institute of Physical Geodesy, Technical University, Graz, Austria


89
Petit, J.-M., Mousis, O., 2004. KBO binaries: how numerous were they? Icarus, 168, 409


